III. WAVE FORCES

The study of wave forces on coastal structures can be classified in two ways: (a) by the type of structure on which the forces act and (b) by the type of wave action against the structure. Fixed coastal structures can generally be classified as one of three types: (a) pile-supported structures such as piers and offshore platforms; (b) wall-type structures such as seawalls, bulkheads, revetments, and some breakwaters; and (c) rubble structures such as many groins, revetments, jetties and breakwaters. Individual structures are often combinations of these three types. The types of waves that can act on these structures are nonbreaking, breaking, or broken waves. Figure 7-66 illustrates the subdivision of wave force problems by structure type and by type of wave action and indicates nine types of force determination problems encountered in design.

![Classification of wave force problems by type of wave action and by structure type.](image)

Rubble structure design does not require differentiation among all three types of wave action; problem types shown as 1R, 2R, and 3R on the figure need consider only nonbreaking and breaking wave design. Horizontal forces on pile-supported structures resulting from broken waves in the surf zone are usually negligible and are not considered. Determination of breaking and nonbreaking wave forces on piles is presented in Section 1 below, Forces on Piles. Nonbreaking, breaking, and broken wave forces on vertical (or nearly vertical) walls are considered in Sections 2, Nonbreaking Wave Forces on Walls, 3, Breaking Wave Forces on Vertical Walls, and 4, Broken Waves. Design of rubble structures is considered in Section 7, Stability of Rubble Structures. NOTE: A careful distinction must be made between the English system use of pounds for weight, meaning force, versus the System International (SI) use of newtons for force. Also, many things measured by their weight (pounds, tons, etc.) in the English system are commonly measured by their mass (kilogram, metric ton, etc.) in countries using the SI system.

7-100
1. Forces on Piles.

   a. Introduction. Frequent use of pile-supported coastal and offshore structures makes the interaction of waves and piles of significant practical importance. The basic problem is to predict forces on a pile due to the wave-associated flow field. Because wave-induced flows are complex, even in the absence of structures, solution of the complex problem of wave forces on piles relies on empirical coefficients to augment theoretical formulations of the problem.

   Variables important in determining forces on circular piles subjected to wave action are shown in Figure 7-67. Variables describing nonbreaking, monochromatic waves are the wave height $H$, water depth $d$, and either wave period $T$, or wavelength $L$. Water particle velocities and accelerations in wave-induced flows directly cause the forces. For vertical piles, the horizontal fluid velocity $u$ and acceleration $du/dt$ and their variation with distance below the free surface are important. The pile diameter $D$ and a dimension describing pile roughness elements $\varepsilon$ are important variables describing the pile. In this discussion, the effect of the pile on the wave-induced flow is assumed negligible. Intuitively, this assumption implies that the pile diameter $D$ must be small with respect to the wavelength $L$. Significant fluid properties include the fluid density $\rho$ and the kinematic viscosity $\nu$. In dimensionless terms, the important variables can be expressed as follows:

   \[
   \frac{H}{\sqrt{gT}} = \text{dimensionless wave steepness}
   \]

   \[
   \frac{d}{\sqrt{gT}} = \text{dimensionless water depth}
   \]

   \[
   \frac{D}{L} = \text{ratio of pile diameter to wavelength (assumed small)}
   \]

   \[
   \frac{\varepsilon}{D} = \text{relative pile roughness}
   \]

   and

   \[
   \frac{HD}{T \nu} = \text{a form of the Reynolds' number}
   \]

   Given the orientation of a pile in the flow field, the total wave force acting on the pile can be expressed as a function of these variables. The variation of force with distance along the pile depends on the mechanism by which the forces arise; that is, how the water particle velocities and accelerations cause the forces. The following analysis relates the local force, acting on a section of pile element of length $dz$, to the local fluid velocity and acceleration that would exist at the center of the pile if the pile were not present. Two dimensionless force coefficients, an inertia or mass coefficient $C_M$ and a drag coefficient $C_D$, are used to establish the wave-force relationships. These coefficients are determined by experimental
measurements of force, velocity, and acceleration or by measurements of force and water surface profiles, with accelerations and velocities inferred by assuming an appropriate wave theory.

The following discussion initially assumes that the force coefficients $C_M$ and $C_D$ are known and illustrates the calculation of forces on vertical cylindrical piles subjected to monochromatic waves. A discussion of the selection of $C_M$ and $C_D$ follows in Section e, Selection of Hydrodynamic Force Coefficients, $C_D$ and $C_M$. Experimental data are available primarily for the interaction of nonbreaking waves and vertical cylindrical piles. Only general guidelines are given for the calculation of forces on noncircular piles.

b. Vertical Cylindrical Piles and Nonbreaking Waves: (Basic Concepts). By analogy to the mechanism by which fluid forces on bodies occur in unidirectional flows, Morison et al. (1950) suggested that the horizontal force per unit length of a vertical cylindrical pile may be expressed by the following (see Fig. 7-67 for definitions):

$$ f = f_i + f_D = C_M \rho \frac{\pi D^2}{4} \frac{du}{dt} + C_D \frac{1}{2} \rho u |u| $$

(7-20)
where

\[ f_i = \text{inertial force per unit length of pile} \]
\[ f_D = \text{drag force per unit length of pile} \]
\[ \rho = \text{density of fluid (1025 kilograms per cubic meter for sea water)} \]
\[ D = \text{diameter of pile} \]
\[ u = \text{horizontal water particle velocity at the axis of the pile (calculated as if the pile were not there)} \]
\[ \frac{du}{dt} = \text{total horizontal water particle acceleration at the axis of the pile, (calculated as if the pile were not there)} \]
\[ C_D = \text{hydrodynamic force coefficient, the "drag" coefficient} \]
\[ C_M = \text{hydrodynamic force coefficient, the "inertia" or "mass" coefficient} \]

The term \( f_i \) is of the form obtained from an analysis of force on a body in an accelerated flow of an ideal nonviscous fluid. The term \( f_D \) is the drag force exerted on a cylinder in a steady flow of a real viscous fluid is proportional to \( u^2 \) and acts in the direction of the velocity \( u \); for flows that change direction this is expressed by writing \( u^2 \) as \( u |u| \). Although these remarks support the soundness of the formulation of the problem as given by equation (7-20), it should be realized that expressing total force by the terms \( f_i \) and \( f_D \) is an assumption justified only if it leads to sufficiently accurate predictions of wave force.

From the definitions of \( u \) and \( \frac{du}{dt} \), given in equation (7-20) as the values of these quantities at the axis of the pile, it is seen that the influence of the pile on the flow field a short distance away from the pile has been neglected. Based on linear wave theory, MacCamy and Fuchs (1954) analyzed theoretically the problem of waves passing a circular cylinder. Their analysis assumes an ideal nonviscous fluid and leads, therefore, to a force having the form of \( f_i \). Their result, however, is valid for all ratios of pile diameter to wavelength, \( \frac{D}{L_A} \), and shows the force to be about proportional to the acceleration \( \frac{du}{dt} \) for small values of \( \frac{D}{L_A} \) (\( L_A \) is the Airy approximation of wavelength). Taking their result as indicative of how small the pile should be for equation (7-20) to apply, the restriction is obtained that

\[ \frac{D}{L_A} < 0.05 \]

(7-21)

Figure 7-68 shows the relative wavelength \( \frac{L_A}{L_o} \) and pressure factor \( K \) versus \( d/gT^2 \) for the Airy wave theory.

**EXAMPLE PROBLEM 16**

GIVEN: A wave with a period of \( T = 5 \) s, and a pile with a diameter \( D = 0.3 \) m (1 ft) in 1.5 m (4.9 ft) of water.
Figure 7-68. Relative wavelength and pressure factor versus \( d/gT^2 \) (Airy wave theory).
FIND: Can equation (7-20) be used to find the forces?

SOLUTION:

\[
\frac{L}{L_o} = \frac{gT^2}{2\pi^2} = \frac{9.8(5)^2}{2\pi} = 39.0 \text{ m (128.0 ft)}
\]

\[
\frac{d}{gT^2} = \frac{1.5}{9.8(5)^2} = 0.0061
\]

which, using Figure 7-68, gives

\[
\frac{L_A}{L_o} = 0.47
\]

\[L_A = 0.47 L_o = 0.47 (39.0) = 18.3 \text{ m (60.0 ft)}
\]

\[
\frac{D}{L_A} = \frac{0.3}{18.3} = 0.016 < 0.05
\]

Since \( D/L_A \) satisfies equation (7-21), force calculations may be based on equation (7-20).

The result of the example problem indicates that the restriction expressed by equation (7-21) will seldom be violated for pile force calculations. However, this restriction is important when calculating forces on dolphins, caissons, and similar large structures that may be considered special cases of piles.

Two typical problems arise in the use of equation (7-20).

1. Given the water depth \( d \), the wave height \( H \), and period \( T \), which wave theory should be used to predict the flow field?

2. For a particular wave condition, what are appropriate values of the coefficients \( C_D \) and \( C_M \)?

c. Calculation of Forces and Moments. It is assumed in this section that the coefficients \( C_D \) and \( C_M \) are known and are constants. (For the selection of \( C_D \) and \( C_M \) see Chapter 7, Section III.1,e, Selection of Hydrodynamic Force Coefficients \( C_D \) and \( C_M \).) To use equation (7-20), assume that the velocity and acceleration fields associated with the design wave can be described by Airy wave theory. With the pile at \( x = 0 \), as shown in Figure 7-67, the equations from Chapter 2 for surface elevation (eq. 2-10), horizontal velocity (eq. 2-13), and acceleration (eq. 2-15), are

\[
\eta = \frac{H}{2} \cos \left( \frac{2\pi t}{T} \right) \quad (7-22)
\]
Introducing these expressions into equation (7-20) gives

\[ u = \frac{H}{2\ L} \frac{gT}{L} \frac{\cosh \left[ \frac{2\pi (z + d)}{L} \right]}{\cosh \left[ \frac{2\pi d}{L} \right]} \cos \left( \frac{2\pi t}{T} \right) \]  

(7-23)

\[ \frac{du}{dt} = \frac{\partial u}{\partial t} = \frac{g\pi H}{L} \frac{\cosh \left[ \frac{2\pi (z + d)}{L} \right]}{\cosh \left[ \frac{2\pi d}{L} \right]} \sin \left( -\frac{2\pi t}{T} \right) \]  

(7-24)

Equations (7-25) and (7-26) show that the two force components vary with elevation on the pile \( z \) and with time \( t \). The inertia force \( f_i \) is maximum for \( \sin (-2\pi t/T) = 1 \), or for \( t = -T/4 \) for Airy wave theory. Since \( t = 0 \) corresponds to the wave crest passing the pile, the inertia force attains its maximum value \( T/4 \) sec before passage of the wave crest. The maximum value of the drag force component \( f_D \) coincides with passage of the wave crest when \( t = 0 \).

Variation in magnitude of the maximum inertia force per unit length of pile with elevation along the pile is, from equation (7-25), identical to the variation of particle acceleration with depth. The maximum value is largest at the surface \( z = 0 \) and decreases with depth. The same is true for the drag force component \( f_D \); however, the decrease with depth is more rapid since the attenuation factor, \( \cosh \left[ \frac{2\pi (z + d)}{L} \right]/\cosh[2\pi d/L] \), is squared. For a quick estimate of the variation of the two force components relative to their respective maxima, the curve labeled \( K = 1/\cosh[2\pi d/L] \) in Figure 7-68 can be used. The ratio of the force at the bottom to the force at the surface is equal to \( K \) for the inertia forces, and to \( K^2 \) for the drag forces.

The design wave will usually be too high for Airy theory to provide an accurate description of the flow field. Nonlinear theories in Chapter 2 showed that wavelength and elevation of wave crest above stillwater level depend on wave steepness and the wave height-water depth ratio. The influence of steepness on crest elevation \( \eta_c \) and wavelength is presented graphically in Figures 7-69 and 7-70. The use of these figures is illustrated by the following examples.

**Example Problem 17**

**Given:** Depth \( d = 4.5 \text{ m} \) (14.8 ft), wave height \( H = 3.0 \text{ m} \) (9.8 ft), and wave period \( T = 10 \text{ s} \).

**Find:** Crest elevation above stillwater level, wavelength, and relative variation of force components along the pile.
Figure 7-69. Ratio of crest elevation above still-water level to wave height.
Figure 7-70. Wavelength correction factor for finite amplitude effects.
SOLUTION: Calculate,

\[ \frac{d}{gT^2} = \frac{4.5}{9.8(10)^2} = 0.0046 \]

\[ \frac{H}{gT^2} = \frac{3.0}{9.8(10)^2} = 0.0031 \]

From Figure 7-68,

\[ L_A = 0.41 \quad L_o = (0.41) \left( \frac{9.8}{2\pi} \right) T^2 = 63.9 \text{ m (209.7 ft)} \]

From Figure 7-69,

\[ \eta_c = 0.85 \quad H = 2.6 \text{ m (8.5 ft)} \]

From Figure 7-70,

\[ L = 1.165 \quad L_A = 1.165 \times 63.9 = 74.4 \text{ m (244.1 ft)} \]

and from Figure 7-68,

\[ K = \frac{f_c}{f_c (z = -d)} = 0.9 \]

\[ K^2 = \frac{f_D}{f_D (z = -d)} = 0.81 \]

Note the large increase in \( \eta_c \) above the Airy estimate of \( H/2 = 1.5 \text{ m (4.9 ft)} \) and the relatively small change of drag and inertia forces along the pile. The wave condition approaches that of a long wave or shallow-water wave.

************** EXAMPLE PROBLEM 18 **************

**GIVEN:** Same wave conditions as preceding problem: \( H = 3.0 \text{ m (9.8 ft)} \) and \( T = 10 \text{ s} \); however, the depth \( d = 30.0 \text{ m (98.4 ft)} \).

**FIND:** Crest elevation above stillwater level, wavelength, and the relative variation of force components along the pile.
SOLUTION: Calculate,

\[ \frac{d}{g T^2} = \frac{30.0}{9.8 (10)^2} = 0.031 \]
\[ \frac{H}{g T^2} = \frac{3.0}{9.8 (10)^2} = 0.0031 \]

From Figure 7-68,

\[ L_A = 0.89 \quad L_O = 0.89 \left( \frac{9.8}{2\pi} \right) T^2 = 138.8 \text{ m (455.4 ft)} \]

From Figure 7-69,

\[ \eta_c = 0.52 H = 0.52(3.0) = 1.6 \text{ m (5.1 ft)} \]

From Figure 7-70,

\[ L = 1.01 L_A = 1.01 (138.8) = 140.2 \text{ m (459.9 ft)} \]

and from Figure 7-68,

\[ K = \frac{f_i (z = -d)}{f_i (z = 0)} = 0.46 \]
\[ K^2 = \frac{f_n (\alpha = -d)}{f_D (z = 0)} = 0.21 \]

Note the large decrease in forces with depth. The wave condition approaches that of a deepwater wave.

For force calculations, an appropriate wave theory should be used to calculate \( u \) and \( du/dt \). Skjelbreia, et al. (1960) have prepared tables based on Stokes' fifth-order wave theory. For a wide variety of given wave conditions (i.e., water depth, wave period, and wave height) these tables may be used to obtain the variation of \( f_i \) and \( f_D \) with time (values are given for time intervals of \( 2\pi t/T = 20^\circ \)) and position along the pile (values given at intervals of 0.1 d). Similar tables based on Dean's numerical stream-function theory (Dean, 1965b) are published in Dean (1974).

For structural design of a single vertical pile, it is often unnecessary to know in detail the distribution of forces along the pile. Total horizontal
force (F) acting on the pile and total moment of forces (M) about the mud line \( z = -d \) are of primary interest. These may be obtained by integration of equation (7-20).

\[
F = \int_{-d}^{0} f_i \, dz + \int_{-d}^{0} f_D \, dz = F_i + F_D \tag{7-27}
\]

\[
M = \int_{-d}^{0} (z+d) f_i \, dz + \int_{-d}^{0} (z + d) f_D \, dz = M_i + M_D \tag{7-28}
\]

In general form these quantities may be written

\[
F_i = C_{D} \frac{\pi D^2}{4} H K_i \tag{7-29}
\]

\[
F_D = C_{D} \frac{1}{2} \rho g D^2 H K_D \tag{7-30}
\]

\[
M_i = C_{M} \frac{\pi D^2}{4} H K_i \, d \, S_i = F_i \, d \, S_i \tag{7-31}
\]

\[
M_D = C_{D} \frac{1}{2} \rho g D^2 K_D \, d \, S_D = F_D \, d \, S_D \tag{7-32}
\]

in which \( C_D \) and \( C_M \) have been assumed constant and where \( K_i \), \( K_D \), \( S_i \), and \( S_D \) are dimensionless. When using Airy theory (eqs. 7-25 and 7-26), the integration indicated in equations (7-27) and (7-28) may be performed if the upper limit of integration is zero instead of \( \eta \). This leads to

\[
K_i = \frac{1}{2} \tanh \left( \frac{2\pi d}{L} \right) \sin \left( -\frac{2\pi t}{T} \right) \tag{7-33}
\]

\[
K_D = \frac{1}{8} \left( 1 + \frac{4\pi d/L}{\sinh \left( 4\pi d/L \right)} \right) \cos \left( \frac{2\pi t}{T} \right) \cos \left( \frac{2\pi t}{T} \right) \tag{7-34}
\]

\[
= \frac{1}{4} n \cos \left( \frac{2\pi t}{T} \right) \cos \left( \frac{2\pi t}{T} \right) \tag{7-34}
\]
where \( n = C_g / C \) has been introduced to simplify the expressions. From equations (7-33) and (7-34), the maximum values of the various force and moment components can be written

\[
S_i = 1 + \frac{1 - \cosh [2\pi d/L]}{(2\pi d/L) \sinh [2\pi d/L]} \\
S_D = \frac{1}{2} + \frac{1}{2n} \left( \frac{1}{2} + \frac{1 - \cosh [4\pi d/L]}{(4\pi d/L) \sinh [4\pi d/L]} \right)
\] (7-35) (7-36)

where \( K_{im} \) and \( K_{Dm} \) according to Airy theory are obtained from equations (7-33) and (7-34) taking \( t = -T/4 \) and \( t = 0 \), respectively and \( S_i \) and \( S_D \) are given by equations (7-35) and (7-36) respectively.

Equations (7-37) through (7-40) are general. Using Dean's stream-function theory (Dean, 1974), the graphs in Figures 7-71 through 7-74 have been prepared and may be used to obtain \( K_{im} \), \( K_{Dm} \), \( S_{im} \), and \( S_{Dm} \). \( S_i \) and \( S_D \) as given in equations (7-35) and (7-36) for Airy theory, are independent of wave phase angle \( \theta \) and thus are equal to the maximum values. For stream-function and other finite amplitude theories, \( S_i \) and \( S_D \) depend on phase angle; Figures 7-73 and 7-74 give maximum values, \( S_{im} \) and \( S_{Dm} \). The degree of nonlinearity of a wave can be described by the ratio of wave height to the breaking height, which can be obtained from Figure 7-75 as illustrated by the following example.

***************EXAMPLE PROBLEM 19***************

**GIVEN:** A design wave \( H = 3.0 \text{ m (9.8 ft)} \) with a period \( T = 8 \text{ s} \) in a depth \( d = 12.0 \text{ m (39.4 ft)} \).

**FIND:** The ratio of wave height to breaking height.

**SOLUTION:** Calculate

\[
\frac{d}{gT^2} = \frac{12.0}{(9.8)(8)^2} = 0.0191
\]

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Figure 7-73. Inertia force moment arm, $S_{lm}$, versus relative depth, $d/gT^2$. (after Dean, 1973)
Figure 7-74. Drag force moment arm, $S_{Dm}$, versus relative depth, $d/gT^2$. 
Figure 7-75. Breaking wave height and regions of validity of various wave theories.
Enter Figure 7-75 with \( \frac{d}{gT^2} = 0.0191 \) to the curve marked Breaking limit and read,

\[
\frac{H_b}{gT^2} = 0.014
\]

Therefore,

\[
H_b = 0.014 \times gT^2 = 0.014(9.8)(8)^2 = 8.8 \text{ m (28.9 ft)}
\]

The ratio of the design wave height to the breaking height is

\[
\frac{H}{H_b} = \frac{3.0}{8.8} = 0.34
\]

By using equations (7-37) through (7-40) with Figures 7-71 through 7-74, the maximum values of the force and moment components can be found. To estimate the maximum total force \( F_m \), Figures 7-76 through 7-79 by Dean (1965a) may be used. The figure to be used is determined by calculating

\[
W = \frac{C_m D}{C_D H}
\]

(7-41)

and the maximum force is calculated by

\[
F_m = \phi_m w C_D H^2 D
\]

(7-42)

where \( \phi_m \) is the coefficient read from the figures. Similarly, the maximum moment \( M_m \) can be determined from Figure 7-80 through 7-83, which are also based on Dean's stream-function theory (Dean, 1965a). The figure to be used is again determined by calculating \( W \) using equation (7-41), and the maximum moment about the mud line \( (z = -d) \) is found from

\[
M_m = \alpha_m w C_D H^2 Dd
\]

(7-43)

where \( \alpha_m \) is the coefficient read from the figures.

Calculation of the maximum force and moment on a vertical cylindrical pile is illustrated by the following examples.
Figure 7-76. Isolines of $\phi_m$ versus $H/gT^2$ and $d/gT^2 \ldots (W = 0.05)$.
Figure 7-79. Isolines of $\phi_m$ versus $H/gT^2$ and $d/gT^2$ ... ($W = 1.0$).
Figure 7-80. Isolines of $a_m$ versus $H/gT^2$ and $d/gT^2$ ... ($W = 0.05$).
Figure 7-81. Isolines of $\alpha_m$ versus $H/gT^2$ and $d/gT^2$ ... ($W = 0.1$).
Figure 7-83. Isolines of $a_m$ versus $H/gT^2$ and $d/gT^2$ ... ($W = 1.0$).
GIVEN: A design wave with height \( H = 3.0 \text{ m} \) (9.8 ft) and period \( T = 10 \text{ s} \) acts on a vertical circular pile with a diameter \( D = 0.3 \text{ m} \) (1 ft) in depth \( d = 4.5 \text{ m} \) (14.8 ft). Assume that \( C_M = 2.0 \) and \( C_D = 0.7 \), and the density of seawater \( p = 1025.2 \text{ kg/m}^3 \) (1.99 slugs/ft\(^3\)). (Selection of \( C_M \) and \( C_D \) is discussed in Section III,1,e.)

FIND: The maximum total horizontal force and the maximum total moment around the mud line of the pile.

SOLUTION: Calculate

\[
\frac{d}{gT^2} = \frac{4.5}{(9.8)(10)^2} = 0.0046
\]

and enter Figure 7-75 to the breaking limit curve and read

\[
\frac{H}{H_b} = 0.0034
\]

Therefore,

\[
H_b = 0.0034 \times gT^2 = 0.00357(9.8)(10)^2 = 3.3 \text{ m} \text{ (10.8 ft)}
\]

and

\[
\frac{H}{H_b} = \frac{3.0}{3.3} = 0.91
\]

From Figures 7-71 and 7-72, using \( d/gT^2 = 0.0046 \) and \( H = 0.91 H_b \), interpolating between curves \( H = H_b \) and \( H = 3/4 H_b \), find:

\[
K_{im} = 0.38
\]

\[
K_{Dm} = 0.71
\]

From equation 7-37:

\[
F_{im} = C_M \rho g \frac{\pi D^2}{4} H K_{im}
\]

\[
F_{im} = (2)(1025.2)(9.8) \frac{\pi(0.3)^2}{4} (3.0)(0.38) = 1619 \text{ N} \text{ (364 lb)}
\]

and from equation (7-38):

\[
F_{DM} = C_D \frac{1}{2} \rho g D H^2 K_{DM}
\]
\[ F_{DM} = (0.7)(0.5)(1025.2)(9.8)(0.3)(3)^2(0.71) = 6,741 \text{ N (1,515 lb)} \]

From equation (7-41), compute
\[ w = \frac{C_m P}{C_D H} = \frac{(2.0)(0.3)}{(0.7)(3)} = 0.29 \]

Interpolation between Figures 7-77 and 7-78 for \( \phi_m \) is required. Calculate
\[ \frac{H}{gT^2} = \frac{3.0}{(9.8)(10)^2} = 0.0031 \]
and recall that
\[ \frac{d}{gT^2} = 0.0046 \]

Find the points on Figures 7-77 and 7-78 corresponding to the computed values of \( H/gT^2 \) and \( d/gT^2 \) and determine \( \phi_m \) (w = 10,047 N/m³ or 64 lb/ft³).

Figure 7-77: \( W = 0.1 \); \( \phi_m = 0.35 \)
Interpolated Value: \( W = 0.29 \); \( \phi_m \approx 0.365 \)
Figure 7-78: \( W = 0.5 \); \( \phi_m \approx 0.38 \)

From equation (7-42), the maximum force is 2
\[ F_m = \phi_m \ wC_D \ H^2 D \]
\[ F_m = 0.365 (10,047)(0.7)(3)^2(0.3) = 6,931 \text{ N (1,558 lb)} \]
say
\[ F_m = 7,000 \text{ N (1,574 lb)} \]

To calculate the inertia moment component, enter Figure 7-73 with
\[ \frac{d}{gT^2} = 0.0046 \]
and \( H = 0.91 H_b \) (interpolate between \( H = H_b \) and \( H = 3/4 H_b \)) to find
\[ S_{im} = 0.82 \]

Similarly, from Figure 7-74 for the drag moment component, determine
\[ S_{Dm} = 1.01 \]

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Therefore from equation (7-39)

\[ M_{im} = F_{im} \ d \ S_{im} = 1619 \ (4.5) \ (0.82) = 5,975 \ N\cdot m \ (4,407 \ ft\cdot lb) \]

and from equation (7-40)

\[ M_{Dm} = F_{Dm} \ d \ S_{Dm} = 6741 \ (4.5) \ (1.01) = 30.6 \ kN\cdot m \ (22,600 \ ft\cdot lb) \]

The value of \( \alpha_m \) is found by interpolation between Figures 7-81 and 7-82 using \( W = 0.29, \ H/gT^2 = 0.0031, \) and \( d/gT^2 = 0.0046. \)

Figure 7-81: \( W = 0.1 ; \ \alpha_m = 0.33 \)
Interpolated Value \( W = 0.29 ; \ \alpha_m \approx 0.34 \)
Figure 7-82: \( W = 0.5 ; \ \alpha_m = 0.35 \)

The maximum total moment about the mud line is found from equation (7-43).

\[ M = \alpha_m \ wC_D H^2 Dd \]

\[ M_m = 0.34 \ (10,047) \ (0.7) \ (3)^2 \ (0.3) \ (4.5) = 29.1 \ kN\cdot m \ (21,500 \ ft\cdot lb) \]

The moment arm, measured from the bottom, is the maximum total moment \( M_m \) divided by the maximum total force \( F_m \); therefore,

\[ \frac{M_m}{F_m} = \frac{29,100}{6,931} = 4.2 \ m \ (13.8 \ ft) \]

If it is assumed that the upper 0.6 m (2 ft) of the bottom material lacks significant strength, or if it is assumed that scour of 0.6 m occurs, the maximum total horizontal force is unchanged, but the lever arm is increased by about 0.6 m. The increased moment can be calculated by increasing the moment arm by 0.6 m and multiplying by the maximum total force. Thus the maximum moment is estimated to be

\[ (M_m) \ 0.6 \ m \ below \ mud \ line = (4.2 + 0.6) \ F_m = 4.8 \ (6,931) = 33.3 \ kN\cdot m \ (24,500 \ ft\cdot lb) \]

********** EXAMPLE PROBLEM 21 **********

**GIVEN:** A design wave with height \( H = 3.0 \ m \ (9.8 \ ft) \) and period \( T = 10 \ s \) acts on a vertical circular pile with a diameter \( D = 0.3 \ m \ (1.0 \ ft) \) in a depth \( d = 30.0 \ m \ (98.4 \ ft) \). Assume \( C_M = 2.0 \) and \( C_D = 1.2. \)

**FIND:** The maximum total horizontal force and the moment around the mud line of the pile.

7-129
SOLUTION: The procedure used is identical to that of the preceding problem. Calculate

$$\frac{d}{gT^2} = \frac{30.0}{(9.8)(10)^2} = 0.031$$

enter Figure 7-75 to the breaking-limit curve and read

$$\frac{H_b}{gT^2} = 0.0205$$

Therefore

$$H_b = 0.0205 \cdot gT^2 = 0.0205 \cdot (9.8) \cdot (10)^2 = 20.1 \text{ m (65.9 ft)}$$

and

$$\frac{H}{H_b} = \frac{3.0}{20.1} = 0.15$$

From Figures 7-71 and 7-72, using \( \frac{d}{gT^2} = 0.031 \) and interpolating between \( H = 0 \) and \( H = \frac{1}{4} H_b \) for \( H = 0.15 H_b \),

$$K_{im} = 0.44$$

$$K_{Dm} = 0.20$$

From equation (7-37),

$$F_{im} = C_M \cdot \rho g \cdot \frac{\pi D^2}{4} \cdot H K_{im}$$

$$F_{im} = 2.0 \cdot (1025.2) \cdot (9.8) \cdot \frac{\pi (0.3)^2}{4} \cdot (3) \cdot (0.44) = 1,875 \text{ N (422 lb)}$$

and from equation (7-38),

$$F_{Dm} = C_D \cdot \frac{1}{2} \cdot \rho g \cdot DH^2 K_{Dm}$$

$$F_{Dm} = 1.2 \cdot (0.5) \cdot (1025.2) \cdot (9.8) \cdot (0.3)^2 \cdot (3) \cdot (0.20) = 3,255 \text{ N (732 lb)}$$

Compute \( W \) from equation (7-41),

$$W = \frac{C_M \cdot D}{C_D \cdot H} = \frac{2.0 \cdot (0.3)}{1.2 \cdot (3)} = 0.17$$

7-130
Interpolation between Figures 7-77 and 7-78 for \( \phi_m \), using \( \frac{d}{gT^2} = 0.031 \) and \( \frac{H}{gT^2} = 0.0031 \), gives

\[
\phi_m = 0.11
\]

From equation (7-42), the maximum total force is

\[
F_m = \phi_m w C D H^2 D
\]

\[
F_m = 0.11 (10,047) (1.2) (3)^2 (0.3) = 3,581 \text{ N (805 lb)}
\]

say

\[
F_m = 3600 \text{ N (809 lb)}
\]

From Figures 7-73 and 7-74, for \( H = 0.15 H_b \),

\[
S_{im} = 0.57
\]

and

\[
S_{Dm} = 0.69
\]

From equation (7-39),

\[
M_{im} = F_{im} d S_{im} = 1,875 (30.0) (0.57) = 32.1 \text{ kN-m (23,700 ft-lb)}
\]

and from equation (7-40),

\[
M_{Dm} = F_{Dm} d S_{Dm} = 3,255 (30.0) (0.69) = 67.4 \text{ kN-m (49,700 ft-lb)}
\]

Interpolation between Figures 7-81 and 7-82 with \( W = 0.16 \) gives

\[
\alpha_m = 0.08
\]

The maximum total moment about the mud line from equation (7-43) is,

\[
M_m = \alpha_m w C D H^2 Dd
\]

\[
M_m = 0.08 (10,047) (1.2) (3)^2 (0.3) (30.0) = 78.1 \text{ kN-m (57,600 ft-lb)}
\]

If calculations show the pile diameter to be too small, noting that \( F_{im} \) is proportional to \( D^2 \) and \( F_{Dm} \) is proportional to \( D \) will allow adjustment of the force for a change in pile diameter. For example, for the same wave conditions and a 0.6-m (2-ft) -diameter pile the forces become

7-131
The new value of $W$ from equation (7-41) is

$$W = \frac{C_M}{C_D} \frac{2 \times 0.6}{1.2(3)} = 0.33$$

and the new values of $\phi_m$ and $\alpha_m$ are

$$\phi_m = 0.15$$

and

$$\alpha_m = 0.10$$

Therefore, from equation (7-42)

$$(F_m)_{0.6 \text{-m diam.}} = \phi_m w C_D H^2 D$$

$$(F_m)_{0.6 \text{-m diam.}} = 0.15 (10,047) (1.2) (3)^2 (0.6) = 9,766 \text{ N (2,195 lb)}$$

and from equation (7-43)

$$(M_m)_{0.6 \text{-m diam.}} = \alpha_m w C_D H^2 Dd$$

$$(M_m)_{0.6 \text{-m diam.}} = 0.10 (10,047) (1.2) (3)^2 (0.6) (30.0) = 195.3 \text{ kN-m (144,100 ft-lb)}$$

---

d. Transverse Forces Due to Eddy Shedding (Lift Forces). In addition to drag and inertia forces that act in the direction of wave advance, transverse forces may arise. Because they are similar to aerodynamic lift force, transverse forces are often termed *lift forces*, although they do not act vertically but perpendicularly to both wave direction and the pile axis.

Transverse forces result from vortex or eddy shedding on the downstream side of a pile: eddies are shed alternately from one side of the pile and then the other, resulting in a laterally oscillating force.

Laird et al. (1960) and Laird (1962) studied transverse forces on rigid and flexible oscillating cylinders. In general, lift forces were found to depend on the dynamic response of the structure. For structures with a natural frequency of vibration about twice the wave frequency, a dynamic coupling between the structure motion and fluid motion occurs, resulting in
large lift forces. Transverse forces have been observed 4.5 times greater than the drag force.

For rigid structures, however, transverse forces equal to the drag force is a reasonable upper limit. *This upper limit pertains only to rigid structures:* larger lift forces can occur when there is dynamic interaction between waves and the structure (for a discussion see Laird (1962)). The design procedure and discussion that follow pertain only to rigid structures.

Chang (1964), in a laboratory investigation, found that eddies are shed at a frequency that is twice the wave frequency. Two eddies were shed after passage of the wave crest (one from each side of the cylinder), and two on the return flow after passage of the trough. The maximum lift force is proportional to the square of the horizontal wave-induced velocity in much the same way as the drag force. Consequently, for design estimates of the lift force, equation (7-44) may be used:

\[ F_L = F_{Lm} \cos 2\theta = C_L \frac{\rho g DH^2 K_{Dm}}{2} \cos 2\theta \]  

(7-44)

where \( F_L \) is the lift force, \( F_{Lm} \) is the maximum lift force, \( \theta = (2\pi x/L - 2\pi t/T) \), and \( C_L \) is an empirical lift coefficient analogous to the drag coefficient in equation (7-38). Chang found that \( C_L \) depends on the Keulegan-Carpenter (1956) number \( \bar{u}_{\text{max}} \frac{T}{D} \), where \( \bar{u}_{\text{max}} \) is the maximum horizontal velocity averaged over the depth. When this number is less than 3, no significant eddy shedding occurs and no lift forces arise. As \( \bar{u}_{\text{max}} \frac{T}{D} \) increases, \( C_L \) increases until it is approximately equal to \( C_D \) (for rigid piles only). Bidde (1970, 1971) investigated the ratio of the maximum lift force to the maximum drag force \( \frac{F_{Lm}}{F_{Dm}} \) which is nearly equal to \( \frac{C_L}{C_D} \) if there is no phase difference between the lift and drag force (this is assumed by equation (7-44)). Figure 7-84 illustrates the dependence of \( \frac{C_L}{C_D} \) on \( \bar{u}_{\text{max}} \frac{T}{D} \). Both Chang and Bidde found little dependence of \( C_L \) on Reynolds number \( R_e = \bar{u}_{\text{max}} \frac{D}{\nu} \) for the ranges of \( R_e \) investigated. The range of \( R_e \) investigated is significantly lower than the range to be anticipated in the field, hence the data presented should be interpreted merely as a guide in estimating \( C_L \) and then \( F_L \).

The use of equation (7-44) and Figure 7-84 to estimate lift forces is illustrated by the following example.

**********EXAMPLE PROBLEM 22**********

**GIVEN:** A design wave with height \( H = 3.0 \text{ m} \) (9.8 ft) and period \( T = 10 \text{ s} \)
Figure 7-84. Variation of $C_L/C_D$ with Keulegan-Carpenter number and $H/g^{1/2}$.
s acts on a vertical circular pile with a diameter \( D = 0.3 \text{ m} \) (1 ft) in a depth \( d = 4.5 \text{ m} \) (14.8 ft). Assume \( C_M = 2.0 \) and \( C_D = 0.7 \).

**FIND:** The maximum transverse (lift) force acting on the pile and the approximate time variation of the transverse force assuming that Airy theory adequately predicts the velocity field. Also estimate the maximum total force.

**SOLUTION:** Calculate,

\[
\frac{H}{gT^2} = \frac{3.0}{(9.8)(10)} = 0.0031
\]

\[
\frac{d}{gT^2} = \frac{4.5}{(9.8)(10)} = 0.0046
\]

and the average Keulegan-Carpenter number \( \bar{u}_{max} \frac{T}{D} \), using the maximum horizontal velocity at the SWL and at the bottom to obtain \( \bar{u}_{max} \). Therefore, from equation (7-23) with \( z = -d \)

\[
\left( \frac{u_{max}}{u_{max}} \right)_{\text{bottom}} = \frac{H}{gT^2} \frac{1}{2} \frac{L_A}{L_A} \cosh \left( \frac{2\pi}{L_A} \frac{d}{L_A} \right)
\]

\[
\left( \frac{u_{max}}{u_{max}} \right)_{\text{bottom}} = \frac{3.0(9.8)(10)}{2(65.5)}(0.90) = 2.0 \text{ m/s} \ (6.6 \text{ ft/s})
\]

where \( L_A \) is found from Figure 7-68 by entering with \( d/gT^2 \) and reading \( L_A/L_o = 2\pi L_A/gT^2 = 0.42 \). Also, \( 1/\cosh [2\pi d/L] \) is the \( K \) value on Figure 7-68. Then, from equation (7-23) with \( z = 0 \),

\[
\left( \frac{u_{max}}{u_{max}} \right)_{\text{SWL}} = \frac{H}{gT^2} \frac{L_A}{L_A}
\]

\[
\left( \frac{u_{max}}{u_{max}} \right)_{\text{SWL}} = \frac{3.0(9.8)(10)}{2(65.5)} = 2.2 \text{ m/s} \ (7.2 \text{ ft/s})
\]

The average velocity is therefore,

\[
\bar{u}_{max} = \frac{1}{2} \left( \left( \frac{u_{max}}{u_{max}} \right)_{\text{bottom}} + \left( \frac{u_{max}}{u_{max}} \right)_{\text{SWL}} \right)
\]

\[
\bar{u}_{max} = \frac{2.0 + 2.2}{2} = 2.1 \text{ m/s} \ (6.9 \text{ ft/s})
\]
and the average Keulegan-Carpenter number is

\[ \frac{\overline{u_{max}} T}{D} = \frac{2.1 \times 10}{0.3} = 70.0 \]

The computed value of \( \overline{u_{max}} T/D \) is well beyond the range of Figure 7-84, and the lift coefficient should be taken to be equal to the drag coefficient (for a rigid structure). Therefore,

\[ C_{Lmax} = C_D = 0.7 \]

From equation (7-44),

\[ F_L = C_L \frac{\rho R}{2} DH^2 K_{Dm} \cos 2\theta = F_{Lm} \cos 2\theta \]

The maximum transverse force \( F_{Lm} \) occurs when \( \cos 2\theta = 1.0 \). Therefore,

\[ F_{Lm} = 0.7 \times \frac{1025 \times 2}{2} \times (9.8) \times (0.3) \times (3)^2 \times (0.71) = 6,741 \text{ N} \ (1,515 \text{ lb}) \]

where \( K_{Dm} \) is found as in the preceding examples. For the example problem the maximum transverse force is equal to the drag force.

Since the inertia component of force is small (preceding example), an estimate of the maximum force can be obtained by vectorially adding the drag and lift forces. Since the drag and lift forces are equal and perpendicular to each other, the maximum force in this case is simply

\[ F_{max} \approx \frac{F_{Lm}}{\cos 45^\circ} = \frac{6,741}{0.707} = 9,535 \text{ N} \ (2,144 \text{ lb}) \]

which occurs about when the crest passes the pile.

The time variation of lift force is given by

\[ F_L = 6,741 \cos 2\theta \]

******************************************************************************

e. **Selection of Hydrodynamic Force Coefficients** \( C_D \) and \( C_M \). Values of \( C_M \), \( C_D \) and safety factors given in the sections that follow are suggested values only. Selection of \( C_M \), \( C_D \) and safety factors for a given design must be dictated by the wave theory used and the purpose of the structure. Values given here are intended for use with the design curves and equations given in preceding sections for preliminary design and for checking design calculations. More accurate calculations require the use of appropriate wave tables such as those of Dean (1974) or Skjelbreia et al. (1960) along with the appropriate \( C_M \) and \( C_D \).
Figure 7-85. Variation of drag coefficient $C_D$ with Reynolds number $R_e$. 

\[ R_e = \frac{u_{\text{max}}D}{\nu} \]
Factors influencing $C_D$. The variation of drag coefficient $C_D$ with Reynolds number $R_e$ for steady flow conditions is shown in Figure 7-85. The Reynolds number is defined by

$$R_e = \frac{uD}{v} \quad (7-45)$$

where

- $u$ = velocity
- $D$ = pile diameter
- $v$ = kinematic viscosity (approximately $1.0 \times 10^{-5}$ ft$^2$/sec for sea water)

Results of steady-state experiments are indicated by dashed lines (Achenbach, 1968). Taking these results, three ranges of $R_e$ exist:

1. **Subcritical**: $R_e < 1 \times 10^5$ where $C_D$ is relatively constant ($\approx 1.2$).
2. **Transitional**: $1 \times 10^5 < R_e < 4 \times 10^5$ where $C_D$ varies.
3. **Supercritical**: $R_e > 4 \times 10^5$ where $C_D$ is relatively constant ($\approx 0.6 - 0.7$).

Thus, depending on the value of the Reynolds number, the results of steady-state experiments show that the value of $C_D$ may change by about a factor of 2.

The steady-flow curves shown in Figure 7-85 show that the values of $R_e$ defining the transitional region vary from investigator to investigator. Generally, the value of $R_e$ at which the transition occurs depends on the roughness of the pile and the ambient level of turbulence in the fluid. A rougher pile will experience the transition at a smaller $R_e$. In the subcritical region, the degree of roughness has an insignificant influence on the value of $C_D$. However, in the supercritical region, the value of $C_D$ increases with increasing surface roughness. The variation of $C_D$ with surface roughness is given in Table 7-4.

The preceding discussion was based on experimental results obtained under steady, unidirectional flow conditions. To apply these results to the unsteady oscillatory flow conditions associated with waves, it is necessary to define a Reynolds number for the wave motion. As equation (7-23) shows, the fluid velocity varies with time and with position along the pile. In principle, an instantaneous value of the Reynolds number could be calculated, and the corresponding value of $C_D$ used. However the accuracy with which $C_D$ is determined hardly justifies such an elaborate procedure.

Keulegan and Carpenter (1956), in a laboratory study of forces on a cylindrical pile in oscillatory flow, found that over most of a wave cycle the value of the drag coefficient remained about constant. Since the maximum value of the drag force occurs when the velocity is a maximum, it seems
Table 7-4. Steady flow drag coefficients for supercritical Reynolds numbers.

<table>
<thead>
<tr>
<th>Surface of 3-Foot-Diameter Cylinder</th>
<th>Average Drag Coefficient $R_e = 1 \times 10^6$ to $6 \times 10^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth (polished)</td>
<td>0.592</td>
</tr>
<tr>
<td>Bitumastic, ¹ glass fiber, and felt wrap</td>
<td>0.61</td>
</tr>
<tr>
<td>Bitumastic, glass fiber, and felt wrap (damaged)</td>
<td>0.66</td>
</tr>
<tr>
<td>Number 16 grit sandpaper (approximately equivalent to a vinyl-mastic coating on a 1- to 2-foot-diameter cylinder)</td>
<td>0.76</td>
</tr>
<tr>
<td>Bitumastic, glass fiber, and burlap wrap (approximately equivalent to bitumastic, glass fiber, and felt wrap on a 1- to 2-foot-diameter cylinder)</td>
<td>0.78</td>
</tr>
<tr>
<td>Bitumastic and oyster shell coating (approximately equivalent to light fouling on a 1- to 2-foot-diameter cylinder)</td>
<td>0.88</td>
</tr>
<tr>
<td>Bitumastic and oyster shell with concrete fragments coating (approximately equivalent to medium barnacle fouling on a 1- to 2-foot-diameter cylinder)</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Blumberg and Rigg, 1961

¹Bitumastic is a composition of asphalt and filler (as asbestos shorts) used chiefly as a protective coating on structural metals exposed to weathering or corrosion.

justified to use the maximum value of the velocity $u_{max}$ when calculating a wave Reynolds number. Furthermore, since the flow near the still-water level contributes most to the moment around the mud line, the location at which $u_{max}$ is determined is chosen to be $z = 0$. Thus, the wave Reynolds number is

$$R_e = \frac{u_{max} D}{\nu} \tag{7-46}$$

where $\nu$ = kinematic viscosity of the fluid $(\nu = 1.0 \times 10^{-5} \text{ ft}^2/\text{s} \text{ for salt water})$ and $u_{max}$ = maximum horizontal velocity at $z = 0$, determined from Airy theory, is given by
The ratio \( \frac{L_A}{L_o} \) can be obtained from Figure 7-68.

An additional parameter, the importance of which was cited by Keulegan and Carpenter (1956), is the ratio of the amplitude of particle motion to pile diameter. Using Airy theory, this ratio \( \frac{A}{D} \) can be related to a period parameter equal to \( \frac{(u_{max} T)}{D} \) (introduced by Keulegan and Carpenter) thus:

\[
\frac{A}{D} = \frac{1}{2\pi} \frac{u_{max}}{D} T \tag{7-48}
\]

When \( z = 0 \) equation (7-48) gives

\[
\frac{A}{D} = \frac{H}{2} \frac{1}{\tanh \left[ \frac{2\pi a}{L_A} \right]} = \frac{H}{2} \frac{L_o}{L_A} \tag{7-49}
\]

The ratio \( \frac{L_A}{L_o} \) is from Figure 7-68.

In a recent laboratory study by Thirriot et al. (1971), it was found that for

\[
\frac{A}{D} > 10 \quad , \quad C_D = C_D \quad (\text{steady flow})
\]

\[
1 < \frac{A}{D} < 10 \quad , \quad C_D > C_D \quad (\text{steady flow})
\]

Combining this with equation (7-49), the steady-state value of \( C_D \) should apply to oscillatory motion, provided

\[
\frac{A}{D} = \frac{H}{2D} \frac{L_o}{L_A} > 10 \tag{7-50}
\]

or equivalently,

\[
\frac{H}{D} > 20 \frac{L_A}{L_o} \tag{7-51}
\]

--------------------------------------------------------

**EXAMPLE PROBLEM 23**

**GIVEN:** A design wave with height of \( H = 3.0 \text{ m} \) (9.8 ft) and period \( T = 10 \text{ s} \) in a depth \( d = 4.5 \text{ m} \) (14.8 ft) acts on a pile of diameter \( D = 0.3 \text{ m} \) (0.9 ft).

**FIND:** Is the condition expressed by the inequality of equation (7-51) satisfied?
SOLUTION: Calculate,

\[ \frac{d}{2gT} = 0.0046 \]

From Figure 7-68:

\[ \frac{L}{L_o} = 0.41 \]

Then,

\[ \frac{H}{D} = \frac{3.0}{0.3} = 10 > 20 \frac{L}{L_o} = 8.2 \]

Therefore, the inequality is satisfied and the steady-state \( C_D \) can be used.

Thirriot, et al. (1971) found that the satisfaction of equation (7-51) was necessary only when \( R_e < 4 \times 10^4 \). For larger Reynolds numbers, they found \( C_D \) approximately equal to the steady flow \( C_D \), regardless of the value of \( A/D \). It is therefore unlikely that the condition imposed by equation (7-51) will be encountered in design. However, it is important to realize the significance of this parameter when interpreting data of small-scale experiments. The average value of all the \( C_D \)'s obtained by Keulegan and Carpenter (1956) is \( (C_D)_{avg} = 1.52 \). The results plotted in Figure 7-85 (Thirriot et al., 1971) that account for the influence of \( A/D \) show that \( C_D = 1.2 \) is a more representative value for the range of Reynolds numbers covered by the experiments.

To obtain experimental values for \( C_D \) for large Reynolds numbers, field experiments are necessary. Such experiments require simultaneous measurement of the surface profile at or near the test pile and the forces acting on the pile. Values of \( C_D \) (and \( C_M \)) obtained from prototype-scale experiments depend critically on the wave theory used to estimate fluid flow fields from measured surface profiles.

**EXAMPLE PROBLEM 24**

**GIVEN:** When the crest of a wave, with \( H = 3.0 \) m (9.8 ft) and \( T = 10 \) s, passes a pile of \( D = 0.3 \) m (0.9 ft) in \( 4.5 \) m (14.8 ft) of water, a force \( F = F_{Dm} = 7000 \) N (1,573 lb) is measured.

**FIND:** The appropriate value of \( C_D \).

**SOLUTION:** From Figure 7-72 as in the problem of the preceding section, \( K_{DM} = 0.71 \) when \( H = 0.87 H_b \). The measured force corresponds to \( F_{Dm} \) therefore, rearranging equation (7-38),

\[ C_D = \frac{F_{Dm}}{(1/2)\rho g D_m^2 K_{DM}} \]
If Airy theory had been used \((H = 0)\), Figure 7-72 with \(d/gT^2 = 0.0046\) would give \(K_{DM} = 0.23\), and therefore

\[
\frac{(C_D)_{Airy}}{(C_D)_{H = 0.87 \frac{H}{b}}} = \frac{K_{Dm}(H = 0.87 \frac{H}{b})}{K_{Dm}(H = 0)} = 0.73 \times \frac{0.71}{0.235} = 2.25
\]

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

**EXAMPLE PROBLEM 25**

**GIVEN:** Same conditions as the preceding example, but with a wave height \(H = 15.0\) m (49.2 ft), a depth \(d = 30.0\) m (98.4 ft), and \(F = F_{Dm} = 130,000\) N (29,225 lb).

**FIND:** The appropriate value of \(C_D\).

**SOLUTION:** From Figure 7-75 \(H_b = 20.6\) m (68 ft); then \(H/H_b = 15.0/20.6 = 0.73\). Entering Figure 7-72 with \(d/gT^2 = 0.031\), \(K_{Dm} = 0.38\) is found. Therefore, from equation (7-33),

\[
C_D = \frac{F_{Dm}}{\frac{1}{2} \rho g H b^2 K_{Dm}} = \frac{130,000}{0.5(1025.2)(9.8)(0.3)(15.0)^2(0.38)} = 1.01
\]

If Airy theory had been used, \(K_{Dm} = 0.17\) and

\[
(C_D)_{Airy} = \frac{(C_D)_{H = 0.73 \frac{H}{b}}}{(C_D)_{Airy (H = 0)}} = \frac{K_{Dm}(H = 0.73 \frac{H}{b})}{K_{Dm}(H = 0)} = (1.01) \times \frac{0.38}{0.17} = 2.26
\]

Some of the difference between the two values of \(C_D\) exists because the SWL (instead of the wave crest) was the upper limit of the integration performed to obtain \(K_{Dm}\) for Airy theory. The remaining difference occurs because Airy theory is unable to describe accurately the water-particle velocities of finite-amplitude waves.

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

7-142
The two examples show the influence of the wave theory used on the value of $C_D$ determined from a field experiment. Since the determination of wave forces is the inverse problem (i.e., $C_D$ and wave conditions known), it is important in force calculations to use a wave theory that is equivalent to the wave theory used to obtain the value of $C_D$ (and $C_M$). A wave theory that accurately describes the fluid motion should be used in the analysis of experimental data to obtain $C_D$ (and $C_M$) and in design calculations.

Results obtained by several investigators for the variation of $C_D$ with Reynolds number are indicated in Figure 7-85. The solid line is generally conservative and is recommended for design along with Figures 7-72 and 7-74 with the Reynolds number defined by equation (7-45).

***************EXAMPLE PROBLEM 26***************

FIND: Were the values of $C_D$ used in the preceding example problems reasonable?

SOLUTION: For the first example with $H = 3.0$ m (9.8 ft), $T = 10$ s, $d = 4.5$ m (14.8 ft), and $D = 0.3$ m (1 ft), from equation (7-47),

$$u_{max} = \frac{\pi H}{T} \frac{L_o}{L_d}$$

$$u_{max} = \frac{\pi 3.0}{10} \frac{1}{0.41} = 2.3 \text{ m (7.5 ft/s)}$$

From equation (7-46)

$$Re = \frac{u_{max} D}{\nu} \quad (\nu = 9.29 \times 10^{-7} \text{ m}^2/\text{s})$$

$$Re = \frac{(2.3)(0.3)}{9.29 \times 10^{-7}} = 7.43 \times 10^5$$

From Figure 7-85, $C_D = 0.7$, which is the value used in the preceding example.

For the example with $H = 3.0$ m (9.8 ft), $T = 10$ s, $d = 30.0$ m (98.4 ft), and $D = 0.3$ m (1 ft), from equation (7-47),

$$u_{max} = \frac{\pi (3.0)(1)}{(10)(0.89)} = 1.1 \text{ m/s (3.6 ft/s)}$$

From equation (7-46),

$$Re = \frac{(1.1)(0.3)}{9.29 \times 10^{-7}} = 3.55 \times 10^5$$

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From Figure 7-85, \( C_D = 0.89 \) which is less than the value of \( C_D = 1.2 \) used in the force calculation. Consequently, the force calculation gave a high force estimate.

Hallermeier (1976) found that when the parameter \( \frac{u^2}{gD} \) is approximately equal to 1.0, the coefficient of drag \( C_D \) may significantly increase because of surface effects. Where this is the case, a detailed analysis of forces should be performed, preferably including physical modeling.

(2). Factors Influencing \( C_M \). MacCamy and Fuchs (1954) found by theory that for small ratios of pile diameter to wavelength,

\[
C_M = 2.0
\]  \hspace{1cm} (7-52)

This is identical to the result obtained for a cylinder in accelerated flow of an ideal or nonviscous fluid (Lamb, 1932). The theoretical prediction of \( C_M \) can only be considered an estimate of this coefficient. The effect of a real viscous fluid, which accounted for the term involving \( C_D \) in equation (7-48), will drastically alter the flow pattern around the cylinder and invalidate the analysis leading to \( C_M = 2.0 \). The factors influencing \( C_D \) also influence the value of \( C_M \).

No quantitative dependence of \( C_M \) on Reynolds number has been established, although Bretschneider (1957) indicated a decrease in \( C_M \) with increasing \( R_e \). However for the range of Reynolds numbers \( (R_e < 3 \times 10^4) \) covered by Keulegan and Carpenter (1956) the value of the parameter \( A/D \) plays an important role in determining \( C_M \). For \( A/D < 1 \) they found \( C_M = 2.0 \). Since for small values of \( A/D \) the flow pattern probably deviates only slightly from the pattern assumed in the theoretical development, the result of \( C_M = 2.0 \) seems reasonable. A similar result was obtained by Jen (1968) who found \( C_M \approx 2.0 \) from experiments when \( A/D < 0.4 \). For larger \( A/D \) values that are closer to actual design conditions, Keulegan and Carpenter found (a) a minimum \( C_M = 0.8 \) for \( A/D 2.5 \) and (b) that \( C_M \) increased from 1.5 to 2.5 for \( 6 < A/D < 20 \).

Just as for \( C_D \), Keulegan and Carpenter showed that \( C_M \) was nearly constant over a large part of the wave period, therefore supporting the initial assumption of constant \( C_M \) and \( C_D \).

Table 7-5 presents values of \( C_M \) reported by various investigators. The importance of considering which wave theory was employed when determining \( C_D \) from field experiments is equally important when dealing with \( C_M \).

Based on the information in Table 7-5, the following choice of \( C_M \) is recommended for use in conjunction with Figures 7-71 and 7-72:

\[
\begin{align*}
C_M &= 2.0 \quad \text{when} \quad R_e < 2.5 \times 10^5 \\
C_M &= 2.5 - \frac{e}{5 \times 10^5} \quad \text{when} \quad 2.5 \times 10^5 < R_e < 5 \times 10^5 \\
C_M &= 1.5 \quad \text{when} \quad R_e > 5 \times 10^5
\end{align*}
\]  \hspace{1cm} (7-53)
with \( R_e \) defined by equation (7-46).

### Table 7-5. Experimentally determined values of \( C_M \)

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Approximate ( R_e )</th>
<th>( C_M ) (^*)</th>
<th>Type of Experiment and Theory Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keulegan and Carpenter (1956)</td>
<td>(&lt;3 \times 10^4)</td>
<td>1.5 to 2.5</td>
<td>Oscillatory laboratory flow ( (A/D &gt; 6) )</td>
</tr>
<tr>
<td>Bretschneider (1957)</td>
<td>(1.6 \times 10^5) to (2.3 \times 10^5)</td>
<td>2.26 to 2.02</td>
<td>Field experiments</td>
</tr>
<tr>
<td></td>
<td>(3.8 \times 10^5) to (6 \times 10^5)</td>
<td>1.74 to 1.23</td>
<td>Linear theory</td>
</tr>
<tr>
<td>Wilson (1965)</td>
<td>large ( (&gt;5 \times 10^5) )</td>
<td>1.53</td>
<td>Field experiment, spectrum</td>
</tr>
<tr>
<td>Skjelbreia (1960)</td>
<td>large ( (&gt;5 \times 10^5) )</td>
<td>1.02 ± 0.53</td>
<td>Field experiments, Stokes' fifth-order theory</td>
</tr>
<tr>
<td>Dean and Aagaard (1970)</td>
<td>(2 \times 10^5) to (2 \times 10^6)</td>
<td>1.2 to 1.7</td>
<td>Field experiments, Stream-function theory</td>
</tr>
<tr>
<td>Evans (1970)</td>
<td>large ( (&gt;5 \times 10^5) )</td>
<td>1.76 ± 1.05</td>
<td>Field experiments, Numerical wave theory or Stokes' fifth-order theory</td>
</tr>
<tr>
<td>Wheeler (1970)</td>
<td>large ( (&gt;5 \times 10^5) )</td>
<td>1.5</td>
<td>Field experiments, Modified spectrum analysis; using ( C_D = 0.6 ) and ( C_M = 1.5 ), the standard deviation of the calculated peak force was 33 percent</td>
</tr>
</tbody>
</table>

\(^*\) Range or mean ± standard deviation.

The values of \( C_M \) given in Table 7-5 show that Skjelbreia (1960), Dean and Aagaard (1970), and Evans (1970) used almost the same experimental data, and yet estimated different values of \( C_M \). The same applies to their determination of \( C_D \), but while the recommended choice of \( C_D \) from Figure 7-85 is generally conservative, from equation (7-53) the recommended choice of \( C_M \) for \( R_e > 5 \times 10^5 \) corresponds approximately to the average of the reported values. This possible lack of conservatism, however, is not significant since the inertia force component is generally smaller than the drag force component for design conditions. From equations (7-37) and (7-38) the ratio of maximum inertia force to maximum drag force becomes

\[
\frac{F_{im}}{F_{Dm}} = \frac{\pi C_M D K_{im}}{2 C_D H K_{Dm}} \tag{7-54}
\]
For example, if $C_M \approx 2 C_D$ and a design wave corresponding to $H/H_b = 0.75$ is assumed, the ratio $F_{im}/F_{Dm}$ may be written (using Figures 7-71 and 7-72) as

$$\frac{F_{im}}{F_{Dm}} \approx \begin{cases} 1.25 \frac{D}{H} & \text{(shallow-water waves)} \\ 5.35 \frac{D}{H} & \text{(deepwater waves)} \end{cases}$$  \hspace{1cm} (7-55)

Since $D/H$ will generally be smaller than unity for a design wave, the inertia-force component will be much smaller than the drag-force component for shallow-water waves and the two force components will be of comparable magnitude only for deepwater waves.

f. Example Problem 27 and Discussion of Choice of a Safety Factor.

* * * * * * * * * * * * * * * * * * * EXAMPLE PROBLEM 27 * * * * * * * * * * * * * * * * * *

**GIVEN:** A design wave, with height $H = 10.0$ m (32.8 ft) and period $T = 12$ s, acts on a pile with diameter $D = 1.25$ m (4.1 ft) in water of depth $d = 26$ m (85 ft).

**FIND:** The wave force on the pile.

**SOLUTION:** Compute

$$\frac{H}{(gT)^2} = \frac{10.0}{(9.8)(12)^2} = 0.0071$$

and

$$\frac{d}{(gT)^2} = \frac{26}{(9.8)(12)^2} = 0.0184$$

From Figure 7-68, for $d/gT^2 = 0.0184$

$$\frac{L_A}{L_o} = 0.76$$

and

$$L_A = 0.76 L_o = 0.76 \frac{gT^2}{2\pi} = 0.76 \frac{(9.8)(12)^2}{2\pi} = 170.7 \text{ m} (559.9 \text{ ft})$$

From Figure 7-69 for $d/gT^2 = 0.0184$,

$$\frac{\eta_c}{H} = 0.68$$

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and, therefore,
\[ \eta_c = 0.68 \times H = 0.68 \times (10.0) = 6.8 \text{ m (22.3 ft)} \]
say
\[ \eta_c = 7 \text{ m (23 ft)} \]

The structure supported by the pile must be 7 m (23 ft) above the still-water line to avoid uplift forces on the superstructure by the given wave.

Calculate, from equation (7-21),
\[ \frac{D}{L_A} = \frac{1.25}{170.7} = 0.0073 < 0.05 \]

Therefore equation (7-20) is valid.

From Figure 7-75,
\[ \frac{H_B}{gT^2} = 0.014 \]

From Figures 7-71 through 7-74,
\[ K_{im} = 0.40 \]
\[ K_{Dm} = 0.35 \]
\[ S_{im} = 0.59 \]
\[ S_{Dm} = 0.79 \]

From equations (7-46) and (7-47),
\[ u_{\text{max}} = \frac{\pi H}{T} \left( \frac{L_0}{L_A} \right) = \frac{\pi (10.0)}{12} \frac{1}{0.76} = 3.4 \text{ m/s (11.1 ft/s)} \]

and
\[ R = \frac{u_{\text{max}}}{v} \left( \frac{3.4 \times (1.25)}{9.29 \times 10^{-7}} \right)^6 = 4.57 \times 10^{-7} \]
From Figure 7-85, 

\[ C_D = 0.7 \]

and from equation (7-53), with \( Re > 5 \times 10^5 \).

\[ C_M = 1.5 \]

Therefore,

\[
F_{im} = C_M \rho g \frac{\pi D^2}{4} H K_{im}
\]

\[
F_{im} = (1.5)(1025.2)(9.8) \frac{\pi(1.25)^2}{4} (10.0)(0.40) = 74.0 \text{ kN (16,700 lb)}
\]

\[
F_{Dm} = C_D \frac{1}{2} \rho g D H^2 K_{Dm}
\]

\[
F_{Dm} = (0.7)(0.5)(1025.2)(9.8)(1.25)(10.0)^2 (0.35) = 153.8 \text{ kN (34,600 lb)}
\]

\[
M_{im} = F_{im} d S_{im} = (74,000)(26)(0.59) = 1,135 \text{ kN-m (0.837 x 10^6 ft-lb)}
\]

\[
M_{Dm} = F_{Dm} d S_{Dm} = (153,800)(26)(0.79) = 3,160 \text{ kN-m (2.33 x 10^6 ft-lb)}
\]

From equation (7-41),

\[
W = \frac{C_M D}{C_D H} = \frac{(1.5)(1.25)}{(0.7)(10.0)} = 0.27
\]

Interpolating between Figures 7-77 and 7-78 with \( H/gT^2 = 0.0075 \) and \( d/gT^2 = 0.0183 \),

\[ \phi_m = 0.20 \]

Therefore, from equation (7-42),

\[
F_m = \phi_m wC_D H^2 D
\]

\[
F_m = (0.20)(10,047)(0.7)(10.0)^2 (1.25) = 175.8 \text{ kN (39,600 lb)}
\]

Interpolating between Figures 7-81 and 7-82 gives

\[ \alpha_m = 0.15 \]

Therefore, from equation (7-43),

\[
M_m = \alpha_m wC_D H^2 D d
\]
\[ M_m = (0.15)(10,407)(0.7)(10.0)^2(1.25)(26) = 3,429 \text{ kN-m (2.529 x 10}^6 \text{ ft-lb)} \]

Before the pile is designed or the foundation analysis is performed, a safety factor is usually applied to calculated forces. It seems pertinent to indicate (Bretschneider, 1965) that the design wave is often a large wave, with little probability of being exceeded during the life of the structure. Also, since the experimentally determined values of \( C_M \) and \( C_D \) show a large scatter, values of \( C_M \) and \( C_D \) could be chosen so that they would rarely be exceeded. Such an approach is quite conservative. For the recommended choice of \( C_M \) and \( C_D \) when used with the generalized graphs, the results of Dean and Aagaard (1970) show that predicted peak force deviated from measured force by at most \( \pm 50 \) percent.

When the design wave is unlikely to occur it is recommended that a safety factor of 1.5 be applied to calculated forces and moments and that this nominal force and moment be used as the basis for structural and foundation design for the pile.

Some design waves may occur frequently. For example, maximum wave height could be limited by the depth at the structure. If the design wave is likely to occur, a large safety factor, say greater than 2, may be applied to account for the uncertainty in \( C_M \) and \( C_D \).

In addition to the safety factor, changes occurring during the expected life of the pile should be considered in design. Such changes as scour at the base of the pile and added pile roughness due to marine growth may be important. For flow conditions corresponding to supercritical Reynolds numbers (Table 7-5), the drag coefficient \( C_D \) will increase with increasing roughness.

The design procedure presented above is a static procedure; forces are calculated and applied to the structure statically. The dynamic nature of forces from wave action must be considered in the design of some offshore structures. When a structure's natural frequency of oscillation is such that a significant amount of energy in the wave spectrum is available at that frequency, the dynamics of the structure must be considered. In addition, stress reversals in structural members subjected to wave forces may cause failure by fatigue. If fatigue problems are anticipated, the safety factor should be increased or allowable stresses should be decreased. Evaluation of these considerations is beyond the scope of this manual.

Corrosion and fouling of piles also require consideration in design. Corrosion decreases the strength of structural members. Consequently, corrosion rates over the useful life of an offshore structure must be estimated and the size of structural members increased accordingly. Watkins (1969) provides some guidance in the selection of corrosion rates of steel in seawater. Fouling of a structural member by marine growth increases (1) the roughness and effective diameter of the member and (2) forces on the member. Guidance on selecting a drag coefficient \( C_D \) can be obtained from Table 7-4. However, the increased diameter must be carried through the entire design procedure to determine forces on a fouled member.
g. Calculation of Forces and Moments on Groups of Vertical Cylindrical Piles. To find the maximum horizontal force and the moment around the mud line for a group of piles supporting a structure, the approach presented in Section III,1,b must be generalized. Figure 7-86 shows an example group of piles subjected to wave action. The design wave concept assumes a two-dimensional (long-crested) wave; hence the x-direction is chosen as the direction of wave propagation. If a reference pile located at \( x = 0 \) is chosen, the x-coordinate of each pile in the group may be determined from

\[
x_n = l_n \cos \alpha_n \tag{7-56}
\]

where the subscript \( n \) refers to a particular pile and \( l_n \) and \( \alpha_n \) are as defined in Figure 7-86. If the distance between any two adjacent piles is large enough, the forces on a single pile will be unaffected by the presence of the other piles. The problem is simply one of finding the maximum force on a series of piles.

In Section III,1,b, the force variation in a single vertical pile as a function of time was found. If the design wave is assumed to be a wave of permanent form (i.e.) one that does not change form as it propagates), the variation of force at a particular point with time is the same as the variation of force with distance at an instant in time. By introducing the phase angle

\[
\theta = \frac{2\pi x}{L} - \frac{2\pi t}{T} \tag{7-57}
\]

where \( L \) is wavelength, the formulas given in Section III,1,c (eqs. (7-25) and (7-26)) for a pile located at \( x = 0 \) may be written in general form by introducing \( \theta \), defined by \( \frac{2\pi x}{L} - \frac{2\pi t}{T} \) in place of \( -\frac{2\pi t}{T} \).

Using tables (Skjelbreia et al., 1960, and Dean, 1974), it is possible to calculate the total horizontal force \( F(x) \) and moment around the mud line \( M(x) \) as a function of distance from the wave crest \( x \). By choosing the location of the reference pile at a certain position \( x = x_r \) relative to the design wave crest the total force, or moment around the mud line, is obtained by summation

\[
\begin{align*}
F_{Total} &= \sum_{n=0}^{N-1} F(x_r + x_n) \\ M_{Total} &= \sum_{n=0}^{N-1} M(x_r + x_n)
\end{align*}
\tag{7-58}
\tag{7-59}
\]

where

\[
\begin{align*}
N &= \text{total number of piles in the group} \\
x_n &= \text{from equation (7-56)} \\
x_r &= \text{location of reference pile relative to wave crest}
\end{align*}
\]
By repeating this procedure for various choices of $x_r$ it is possible to determine the maximum horizontal force and moment around the mud line for the pile group.

$F_D(\theta)$ is an even function, and $F_i(\theta)$ is an odd function; hence

$$F_D(\theta) = F_D(-\theta) \quad (7-60)$$

and

$$F_i(\theta) = - F_i(-\theta) \quad (7-61)$$

and calculations need only be done for $0 \leq \theta \leq \pi$ radians. Equations (7-60) and (7-61) are true for any wave that is symmetric about its crest, and are therefore applicable if the wave tables of Skjelbreia et al. (1960) and Dean (1974) are used. When these tables are used, the wavelength computed from the appropriate finite amplitude theory should be used to transform $\phi$ into distance from the wave crest, $x$.

The procedure is illustrated by the following examples. For simplicity, Airy theory is used and only maximum horizontal force is considered. The same computation procedure is used for calculating maximum moment.
EXAMPLE PROBLEM 28

GIVEN: A design wave with height $H = 10.0\ m$ (32.8 ft) and period $T = 12\ s$ in a depth $d = 26.0\ (85.3\ ft)$ acts on a pile with a diameter $D = 1.25\ m\ (4.1\ ft)$. (Assume Airy theory to be valid.)

FIND: The variation of the total force on the pile as a function of distance from the wave crest.

SOLUTION: From an analysis similar to that in Section III,1,e,

$$C_D = 0.7$$

and

$$C_M = 1.5$$

From Figures 7-71 and 7-72, using the curve for Airy theory with

$$K_{im} = 0.38;\ K_{Dm} = 0.20$$

and from equations (7-37) and (7-38),

$$F_{im} = 1.5\ (1025.2)\ (9.8)\ \frac{\pi(1.25)^2}{4}\ (10.0)(0.38) = 70.3\ kN\ (15,800\ lb)$$

$$F_{Dm} = 0.7\ (0.5)(1025.2)(9.8)(1.25)(10.0)^2(0.20) = 87.9\ kN\ (19,800\ lb)$$

Combining equations (7-29) and (7-33) gives

$$F_i = F_{im}\ \sin\ \theta$$

and combining equations (7-30) and (7-34) gives

$$F_D = F_{Dm}\ \cos\ \theta|\ \cos\ \theta|$$

where

$$\theta = \frac{2\pi x}{L} - \frac{2\pi t}{T}$$

The wavelength can be found from Figure 7-68,

$$L = L_A = 171\ m$$
From Table 7-6, the maximum force on the example pile occurs when \((20^\circ < \theta < 40^\circ)\); 
\(F_m = 102\) kN (22,930 lb).

Table 7-6. Example calculation of wave force variation with phase angle.

<table>
<thead>
<tr>
<th>(\theta) (deg)</th>
<th>(x) (m)</th>
<th>(F_i = F_{im} \sin \theta) (kN)</th>
<th>(F_D = F_{Dm} \cos \theta \cos \theta) (kN)</th>
<th>(F(\theta) = F_i + F_D) (kN)</th>
<th>(F(-\theta) = F_D - F_i) (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>87.9</td>
<td>87.9</td>
<td>87.9</td>
</tr>
<tr>
<td>20</td>
<td>9.5</td>
<td>24.1</td>
<td>77.6</td>
<td>101.7</td>
<td>53.5</td>
</tr>
<tr>
<td>40</td>
<td>19.0</td>
<td>45.2</td>
<td>51.6</td>
<td>96.8</td>
<td>6.4</td>
</tr>
<tr>
<td>60</td>
<td>28.5</td>
<td>60.9</td>
<td>22.0</td>
<td>82.9</td>
<td>-38.9</td>
</tr>
<tr>
<td>80</td>
<td>38.0</td>
<td>69.2</td>
<td>2.7</td>
<td>71.9</td>
<td>-66.5</td>
</tr>
<tr>
<td>100</td>
<td>47.5</td>
<td>69.2</td>
<td>-2.7</td>
<td>66.5</td>
<td>-71.9</td>
</tr>
<tr>
<td>120</td>
<td>57.0</td>
<td>60.9</td>
<td>-22.0</td>
<td>38.9</td>
<td>-82.9</td>
</tr>
<tr>
<td>140</td>
<td>66.5</td>
<td>45.2</td>
<td>-51.6</td>
<td>6.4</td>
<td>-96.8</td>
</tr>
<tr>
<td>160</td>
<td>76.0</td>
<td>24.1</td>
<td>-77.6</td>
<td>-53.5</td>
<td>-101.7</td>
</tr>
<tr>
<td>180</td>
<td>85.5</td>
<td>0</td>
<td>-87.9</td>
<td>-87.9</td>
<td>-87.9</td>
</tr>
</tbody>
</table>

Note: 1 Newton (N) = 0.225 pounds of force.
1 kN = 1000 N.

EXAMPLE PROBLEM 29

GIVEN: Two piles each with a diameter \(D = 1.25\) m (4.1 ft) spaced 30.0 m (98.4 ft) apart are acted on by a design wave having a height \(H = 10.0\) m (32.8 ft) and a period \(T = 12\) s in a depth \(d = 26\) m (85 ft). The direction of wave approach makes an angle of \(30^\circ\) with a line joining the pile centers.

FIND: The maximum horizontal force experienced by the pile group and the location of the reference pile with respect to the wave crest (phase angle) when the maximum force occurs.

SOLUTION: The variation of total force on a single pile with phase angle \(\theta\) was computed from Airy theory for the preceding problem and is given in Table 7-6. Values in Table 7-6 will be used to compute the maximum horizontal force on the two-pile group. Compute the phase difference between the two piles by equation (7-56)

\[
x_h = l_n \cos \alpha_n = 30 (\cos 30^\circ)
\]

\[
x_h = 26.0 \text{ m (85.2 ft)}
\]
From the previous example problem, \( L = L_A = 171 \text{ m} \) for \( d = 26 \text{ m} \) and \( T = 12 \text{ s} \). Then, from the expression \( x/L = \theta_n/2\pi \),

\[
\theta_n = \frac{2\pi x}{L} = \frac{2\pi(26.0)}{171} = 0.96 \text{ rad}
\]

or

\[
\theta_n = \frac{360^\circ (26.0)}{171} = 54.7^\circ
\]

Values in Table 7-6 can be shifted by 55 degrees and represent the variation of force on the second pile with the phase angle. The total horizontal force is the sum of the two individual pile forces. The same procedure can be applied for any number of piles. Table 7-6 can be used by offsetting the force values by an amount equal to 55 degrees (preferably by a graphical method). The procedure is also applicable to moment computations.

The maximum force is about 183.0 kN when the wave crest is about 8 degrees or \( \left( \frac{8^\circ}{360^\circ} \right) 171 \approx 3.5 \text{ m} (11.5 \text{ ft}) \) in front of the reference pile.

Because Airy theory does not accurately describe the flow field of finite-amplitude waves, a correction to the computed maximum force as determined above could be applied. This correction factor for structures of minor importance might be taken as the ratio of maximum total force on a single pile for an appropriate finite-amplitude theory to maximum total force on the same pile as computed by Airy theory. For example, the forces on a single pile are (from preceding example problems),

\[
(F_{m})_{\text{finite-amplitude}} = 175.9 \text{ kN} \ (39,600 \text{ lb})
\]

\[
(F_{m})_{\text{Airy}} = 102 \text{ kN} \ (22,930 \text{ lb})
\]

Therefore, the total force on the two-pile group, corrected for the finite-amplitude design wave, is given by,

\[
\left[ F_{\text{Total}} \right]_{\text{2 piles}} = \frac{(F_{m})_{\text{finite-amplitude}}}{(F_{m})_{\text{Airy}}} \left[ F_{\text{Total}} \right]_{\text{2 piles}} \quad \text{(computed from Airy theory)}
\]

\[
\left[ F_{\text{Total}} \right]_{\text{2 piles}} = \frac{175.9}{102.0} (183.0) = 315.6 \text{ kN} \ (71,000 \text{ lb})
\]
This approach is an approximation and should be limited to rough calculations for checking purposes only. The use of tables of finite-amplitude wave properties (Skjelbreia et al., 1960 and Dean, 1974) is recommended for design calculations.

As the distance between piles becomes small relative to the wavelength, maximum forces and moments on pile groups may be conservatively estimated by adding the maximum forces or moments on each pile.

The assumption that piles are unaffected by neighboring piles is not valid when distance between piles is less than three times the pile diameter. A few investigations evaluating the effects of nearby piles are summarized by Dean and Harleman (1966).

h. Calculation of Forces on a Nonvertical Cylindrical Pile. A single, nonvertical pile subjected to the action of a two-dimensional design wave traveling in the +x direction is shown in Figure 7-67. Since forces are perpendicular to the pile axis, it is reasonable to calculate forces by equation (7-20) using components of velocity and acceleration perpendicular to the pile. Experiments (Bursnall and Loftin, 1951) indicate this approach may not be conservative, since the drag force component depends on resultant velocity rather than on the velocity component perpendicular to the pile axis. To consider these experimental observations, the following procedure is recommended for calculating forces on nonvertical piles.

For a given location on the pile \((x_o, y_o, z_o)\) in Figure 7-67, the force per unit length of pile is taken as the horizontal force per unit length of a fictitious vertical pile at the same location.

**EXAMPLE PROBLEM 30**

**GIVEN:** A pile with diameter \(D = 1.25\) m (4.1 ft) at an angle of 45 degrees with the horizontal in the x-z plane is acted upon by a design wave with height \(H = 10.0\) m (32.8 ft) and period \(T = 12\) s in a depth \(d = 26\) m (85 ft)

**FIND:** The maximum force per unit length on the pile 9.0 m (29.5 ft) below the SWL (\(z = -9.0\) m).

**SOLUTION:** For simplicity, Airy theory is used. From preceding examples, \(C_M = 1.5\), \(C_D = 0.7\), and \(L = L_A = 171\) m.

From equation (7-25) with \(\sin(-2\pi/T) = 1.0\),

\[
f_{izm} = C_M \rho g \frac{\pi D^2}{4} H \left( \frac{\pi}{L} \cosh \left[ \frac{2\pi(d + z)/L}{\cosh 2\pi d/L} \right] \right)
\]

\[
f_{izm} = 1.5 \left( 1025.2 \right) \left( 9.8 \right) \frac{\pi(1.25)^2}{4} \left( 10.0 \right) \frac{\pi}{171} (0.8) = 2,718 \text{ N/m (186 lb/ft)}
\]
From equation (7-26) with \( \cos (2\pi t/T) = 1.0 \),

\[
f_{Dm} = C_D \frac{\rho g}{2} \frac{H^2}{D} \frac{T^2}{2} \left( \frac{\cosh [2\pi (d + z)/L]}{\cosh [2\pi d/L]} \right)^2
\]

\[
f_{Dm} = 0.7 \frac{(1025.2)(9.8)}{2} \frac{(1.25)(10.0)^2}{4(171)^2} \frac{(9.8)(12)^2}{(0.8)^2} = 3,394 \text{ N/m}
\]

\( = (233 \text{ lb/ft}) \)

The maximum force can be assumed to be given by

\[
f_m = f_{Dm} \frac{F_m}{F_{Dm}}
\]

where \( F_m \) and \( F_{Dm} \) are given by equations (7-42) and (7-38). Substituting these equations into the above gives

\[
f_m = f_{Dm} \frac{\phi_m w C_D H^2}{\rho g/2 DK_{Dm}}
\]

\[
= f_{Dm} \frac{2\phi_m}{K_{Dm}}
\]
From equation (7-41),

\[ W = \frac{C_m D}{C_D H} = \frac{1.5(1.25)}{0.7(10.0)} = 0.27 \]

Interpolating between Figures 7-77 and 7-78 with \( H/gT^2 = 0.0075 \) and \( d/gT^2 = 0.0184 \), it is found that \( \phi_m = 0.20 \).

From a preceding problem,

\[ \frac{H}{H_b} = 0.52 \]

Enter Figure 7-72 with \( d/gT^2 = 0.0183 \) and, using the curve labeled \( 1/2 H_b \), read

\[ K_{Dm} = 0.35 \]

Therefore,

\[ f_m = f_{Dm} K_{Dm} \]

\[ f_m = 3,394.1 \cdot \frac{2(0.2)}{0.35} = 3,879 \text{ N/m (266 lb/ft)} \]

say

\[ f_m = 3,900 \text{ N/m (267 lb/ft)} \]

The maximum horizontal force per unit length at \( z = -9.0 \text{ m (-29.5 ft)} \) on the fictitious vertical pile is \( f_m = 3,900 \text{ N/m} \). This is also taken as the maximum force per unit length perpendicular to the actual inclined pile.

---

**i. Calculation of Forces and Moments on Cylindrical Piles Due to Breaking Waves.**

Forces and moments on vertical cylindrical piles due to breaking waves can, in principle, be calculated by a procedure similar to that outlined in Section III, 1,b by using the generalized graphs with \( H = H_b \). This approach is recommended for waves breaking in deep water (see Ch. 2, Sec. VI, BREAKING WAVES).

For waves in shallow water, the inertia force component is small compared to the drag force component. The force on a pile is therefore approximately

\[ F_m \sim F_{Dm} = C_D \frac{1}{2} \rho g D^2 H^2 K_{Dm} \quad (7-62) \]
Figure 7-72, for shallow-water waves with $H = H_b$, gives $K_{Dm} = 0.96 = 1.0$; consequently the total force may be written

$$F_m = C_D \frac{1}{2} \rho g D H_b^2$$  \hspace{1cm} (7-63)

From Figure 7-74, the corresponding lever arm is $d_b S_{DM} \approx d_b (1.11)$ and the moment about the mud line becomes

$$M_m = F_m (1.11 d_b)$$  \hspace{1cm} (7-64)

Small-scale experiments ($R_e \approx 5 \times 10^4$ by Hall, 1958) indicate that

$$F_m \approx 1.5 \rho g D H_b^2$$  \hspace{1cm} (7-65)

and

$$M_m = H_b$$  \hspace{1cm} (7-66)

Comparison of equation (7-63) with equation (7-65) shows that the two equations are identical if $C_D = 3.0$. This value of $C_D$ is 2.5 times the value obtained from Figure 7-85 ($C_D = 1.2$ for $R_e 5 \times 10^4$). From Chapter 2, Section VI, since $H_b$ generally is smaller than $(1.11) d_b$, it is conservative to assume the breaker height approximately equal to the lever arm, $1.11 d_b$. Thus, the procedure outlined in Section III.1.b of this chapter may also be used for breaking waves in shallow water. However, $C_D$ should be the value obtained from Figure 7-85 and multiplied by 2.5.

Since the Reynolds number generally will be in the supercritical region, where according to Figure 7-85, $C_D = 0.7$, it is recommended to calculate breaking wave forces using

$$(C_D)^{breaking} = 2.5 (0.7) = 1.75$$  \hspace{1cm} (7-67)

The above recommendation is based on limited information; however, large-scale experiments by Ross (1959) partially support its validity.

For shallow-water waves near breaking, the velocity near the crest approaches the celerity of wave propagation. Thus, as a first approximation the horizontal velocity near the breaker crest is

$$u_{crest} = \sqrt{g d_b} = \sqrt{g H_b}$$  \hspace{1cm} (7-68)

where $H_b$ is taken approximately equal to $d_b$, the depth at breaking. Using
equation (7-68) for the horizontal velocity, and taking \( C_D = 1.75 \), the force per unit length of pile near the breaker crest becomes

\[
f_{Dm} = C_D \frac{1}{2} \rho h u_c^2 \approx 0.88 \rho g D H_b
\]

Table 7-7 is a comparison between the result calculated from equation (7-69) with measurements by Ross (1959) on a 1-foot-diameter pile \( (R_e \approx 1.3 \times 10^6) \).

Table 7-7. Comparison of measured and calculated breaker force.\(^1\)

<table>
<thead>
<tr>
<th>Breaker Height m (ft)</th>
<th>( f_{Dm}^2 ) N/m (lb/ft)</th>
<th>( f_{Dm}^3 ) N/m (lb/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 (3.7)</td>
<td>3021 (207)</td>
<td>3211 (220)</td>
</tr>
<tr>
<td>1.16 (3.8)</td>
<td>3108 (213)</td>
<td>3648 (250)</td>
</tr>
<tr>
<td>1.2 (4.1)</td>
<td>3357 (230)</td>
<td>1824 (125)</td>
</tr>
<tr>
<td>1.3 (4.2)</td>
<td>3430 (235)</td>
<td>2481 (170)</td>
</tr>
<tr>
<td>1.3 (4.2)</td>
<td>3430 (235)</td>
<td>4086 (280)</td>
</tr>
<tr>
<td>1.5 (4.9)</td>
<td>4013 (275)</td>
<td>3648 (250)</td>
</tr>
</tbody>
</table>

1 Values given are force per unit length of pile near breaker crest.
2 Calculated from equation (7-69).
3 Measured by Ross, 1959.

Based on this comparison, the choice of \( C_D = 1.75 \) for \( R_e > 5 \times 10^5 \) appears justified for calculating forces and moments due to breaking waves in shallow water.

(a) Calculation of Forces on Noncircular Piles. The basic force equation (eq. 7-20) can be generalized for piles of other than those with a circular cross section, if the following substitutions are made:

\[
\frac{\pi D^2}{4} = \text{volume per unit length of pile}
\]

(7-70)

where

\[ D = \text{width perpendicular to flow direction per unit length of pile} \]
Substituting the above quantities for a given noncircular pile cross section, equation (7-20) may be used. The coefficients $K_{im}$, etc., depend only on the flow field and are independent of pile cross-section geometry; therefore, the generalized graphs are still valid. However, the hydrodynamic coefficients $C_D$ and $C_M$ depend strongly on the cross-section shape of the pile. If values for $C_D$ and $C_M$ corresponding to the type of pile to be used are available, the procedure is identical to the one presented in previous sections.

Keulegan and Carpenter (1956) performed tests on flat plate in oscillating flows. Equation (7-20) in the form applicable for a circular cylinder, with $D$ taken equal to the width of the plate, gave

\[
\begin{align*}
3 &< C_M < 4.5 \\
1.8 &< C_D < 2.7
\end{align*}
\quad \text{for } \frac{A}{D} > 10
\tag{7-71}
\]

The fact that $C_D$ approaches the value of 1.8 as $A/D$ (eq. 7-50) increases is in good agreement with results obtained under steady flow conditions (Rouse, 1950).

The following procedure is proposed for estimating forces on piles having sharp-edged cross sections for which no empirical data are available for values of $C_M$ and $C_D$.

1. The width of the pile measured perpendicular to the flow direction is assumed to be the diameter of an equivalent circular cylindrical pile, $D$.
2. The procedures outlined in the preceding sections are valid, and the formulas are used as if the pile were of circular cross section with diameter $D$.
3. The hydrodynamic coefficients are chosen within the range given by equation (7-71); i.e., $C_M \approx 3.5$ and $C_D \approx 2.0$.

This approach is approximate and should be used with caution. More accurate analyses require empirical determination of $C_M$ and $C_D$ for the pile geometry under consideration.

Forces resulting from action of broken waves on piles are much smaller than forces due to breaking waves. When pile-supported structures are constructed in the surf zone, lateral forces from the largest wave breaking on the pile should be used for design (see Sec. I,2). While breaking-wave forces in the surf zone are great per unit length of pile, the pile length actually subjected to wave action is usually short, hence results in a small total force. Pile design in this region is usually governed primarily by vertical loads acting along the pile axis.
2. Nonbreaking Wave Forces on Walls.

a. General. In an analysis of wave forces on structures, a distinction is made between the action of nonbreaking, breaking, and broken waves (see Sec. 1.2, Selection of Design Wave). Forces due to nonbreaking waves are primarily hydrostatic. Broken and breaking waves exert an additional force due to the dynamic effects of turbulent water and the compression of entrapped air pockets. Dynamic forces may be much greater than hydrostatic forces; therefore, structures located where waves break are designed for greater forces than those exposed only to nonbreaking waves.

b. Nonbreaking Waves. Typically, shore structures are located in depths where waves will break against them. However, in protected regions, or where the fetch is limited, and when depth at the structure is greater than about 1.5 times the maximum expected wave height, nonbreaking waves may occur.

Sainflou (1928) proposed a method for determining the pressure due to nonbreaking waves. The advantage of his method has been ease of application, since the resulting pressure distribution may be reasonably approximated by a straight line. Experimental observations by Rundgren (1958) have indicated Sainflou's method overestimates the nonbreaking wave force for steep waves. The higher order theory by Miche (1944), as modified by Rundgren (1958), to consider the wave reflection coefficient of the structure, appears to best fit experimentally measured forces on vertical walls for steep waves, while Sainflou's theory gives better results for long waves of low steepness. Design curves presented here have been developed from the Miche-Rundgren equations and the Sainflou equations.

c. Miche-Rundgren: Nonbreaking Wave Forces. Wave conditions at a structure and seaward of a structure (when no reflected waves are shown) are depicted in Figure 7-88. The wave height that would exist at the structure if the structure were not present is the incident wave height \( H_i \). The wave height that actually exists at the structure is the sum of \( H_i \) and the height of the wave reflected by the structure \( H_r \). The wave reflection coefficient \( \chi \) equals \( H_r / H_i \). Wave height at the wall \( H_w \) is given as

\[
H_w = H_i + H_r (1 + \chi) H_i
\]

(7-72)

If reflection is complete and the reflected wave has the same amplitude as the incident wave, then \( \chi = 1 \) and the height of the clapotis or standing wave at the structure will be \( 2H_i \). (See Figure 7-88 for definition of terms associated with a clapotis at a vertical wall.) The height of the clapotis crest above the bottom is given by

\[
y_c = d + h_o + \frac{1 + \chi}{2} H_i
\]

(7-73)

where \( h_o \) is the height of the clapotis orbit center above SWL.

The height of the clapotis trough above the bottom is given by

\[
y_t = d + h_o - \frac{1 + \chi}{2} H_i
\]

(7-74)
The reflection coefficient, and consequently clapotis height and wave force, depends on the geometry and roughness of the reflecting wall and possibly on wave steepness and the "wave height-to-water depth" ratio. Domzig (1955) and Greslou and Mahe (1954) have shown that the reflection coefficient decreases with both increasing wave steepness and "wave height-to-water depth" ratio. Goda and Abe (1968) indicate that for reflection from smooth vertical walls this effect may be due to measurement techniques and could be only an apparent effect. Until additional research is available, it should be assumed that smooth vertical walls completely reflect incident waves and $\chi = 1$. Where wales, tiebacks, or other structural elements increase the surface roughness of the wall by retarding vertical motion of the water, a lower value of $\chi$ may be used. A lower value of $\chi$ also may be assumed when the wall is built on a rubble base or when rubble has been placed seaward of the structure toe. 

Any value of $\chi$ less than 0.9 should not be used for design purposes.

Pressure distributions of the crest and trough of a clapotis at a vertical wall are shown in Figure 7-89. When the crest is at the wall, pressure
increases from zero at the free water surface to \( w d + p_l \) at the bottom, where \( p_l \) is approximated as

\[
p_l = \left( \frac{1 + \chi}{2} \right) \frac{w H_c}{\cosh (2\pi d/L)}
\]  

(7-75)

Figure 7-89. Pressure distributions for nonbreaking waves.

When the trough is at the wall, pressure increases from zero at the water surface to \( w d - p_l \) at the bottom. The approximate magnitude of wave force may be found if the pressure is assumed to increase linearly from the free surface to the bottom when either the crest or trough is at the wall. However, this estimate will be conservative by as much as 50 percent for steep waves near the breaking limit.

Figures 7-90 through 7-95 permit a more accurate determination of forces and moments resulting from a nonbreaking wave at a wall. Figures 7-90 and 7-92 show the dimensionless height of the clapotis orbit center above stillwater level, dimensionless horizontal force due to the wave, and dimensionless moment about the bottom of the wall (due to the wave) for a reflection coefficient \( \chi = 1 \). Figures 7-93 through 7-95 represent identical dimensionless parameters for \( \chi = 0.9 \).

The forces and moments found by using these curves do not include the force and moment due to the hydrostatic pressure at still-water level (see Figure 7-89).
Figure 7-90. Nonbreaking waves; \( \chi = 1.0 \) .
Figure 7-91. Nonbreaking wave forces; $x = 1.0$.
Figure 7-92. Nonbreaking wave moment; $\chi = 1.0$. 

Hydrostatic Moment Not Included

$M_{wd^3}$

$H_i/d = 0.1$

$H_i/d = 0.7$

Envelope to Observed Breaking Waves

Wave Crest at Structure

Wave Trough at Structure
Figure 7-93. Nonbreaking waves; $\chi = 0.9$. 
Figure 7-94. Nonbreaking wave forces; $\chi = 0.9$. 
Figure 7-95. Nonbreaking wave moment; \( \lambda = 0.9 \).
When it is necessary to include the hydrostatic effects (e.g., seawalls), the total force and moment are found by the expressions

\[
F_{\text{total}} = \frac{wd^2}{2} + F_{\text{wave}}
\]

\[
M_{\text{total}} = \frac{wd^3}{6} + M_{\text{wave}}
\]

where \( F_{\text{wave}} \) and \( M_{\text{wave}} \) are found from the design curves. The use of the figures to determine forces and moments is illustrated in the following example.

**EXAMPLE PROBLEM 31**

**GIVEN:**

(a) Smooth-faced vertical wall \((\chi = 1.0)\).

(b) Wave height at the structure if the structure were not there \(H_i = 1.5 \text{ m (5 ft)}\).

(c) Depth at structure \(d = 3 \text{ m (10 ft)}\).

(d) Range of wave periods to be considered in design \(T = 6 \text{ s (minimum)} \text{ or } T = 10 \text{ s (maximum)}\).

**FIND:** The nonbreaking wave force and moments against a vertical wall resulting from the given wave conditions.

**SOLUTION:** Details of the computations are given for only the 6-second wave. From the given information, compute \(H_i/d\) and \(H_i/gT^2\) for the design condition:

\[
\frac{H_i}{d} = \frac{1.5}{3} = 0.5 , \quad \frac{H_i}{gT^2} = \frac{1.5}{9.81 (6)^2} = 0.0043 \quad (T = 6 \text{ s})
\]

Enter Figure 7-90 (because the wall is smooth) with the computed value of \(H_i/gT^2\), and determine the value of \(H_o/H_i\) from the curve for \(H_i/d = 0.5\). (If the wave characteristics fall outside of the dashed line) the structure will be subjected to breaking or broken waves and the method for calculating breaking wave forces should be used.)

For

\[
\frac{H_i}{gT^2} = 0.0043 \quad \frac{h}{H_i} = 0.66 \quad (T = 6 \text{ s})
\]

Therefore,

\[
h_o = 0.70 (H_i) = 0.66 (1.5) = 1.00 \text{ m (3.3 ft)} \quad (T = 6 \text{ s})
\]
The height of the free surface above the bottom \( y \), when the wave crest and trough are at the structure, may be determined from equations (7-73) and (7-74) as follows:

\[
y_c = d + h_o + \left(\frac{1 + \chi}{2}\right) H_i^2
\]

and

\[
y_t = d + h_o - \left(\frac{1 + \chi}{2}\right) H_i^2
\]

\[
y_c = 3 + 1.00 + (1)(1.5) = 5.50 \text{ m (18.1 ft)}
\]

\[
y_t = 3 + 1.00 - (1)(1.5) = 2.50 \text{ m (8.2 ft) (T = 6 s)}
\]

A similar analysis for the 10-second wave gives

\[
y_c = 5.85 \text{ m (19.2 ft)}
\]

\[
y_t = 2.85 \text{ m (9.4 ft) (T = 6 s)}
\]

The wall would have to be about 6 meters (20 feet) high if it were not to be overtopped by a 1.5-meter-(5-foot-) high wave having a period of 10 seconds.

The horizontal wave forces may be evaluated using Figure 7-91. Entering the figure with the computed value of \( H_i/gT^2 \), the value of \( F/wd^2 \) can be determined from either of two curves of constant \( H_i/d \). The upper family of curves (above \( F/wd^2 = 0 \)) will give the dimensionless force when the crest is at the wall: \( F_c/wd^2 \); the lower family of curves (below \( F/wd^2 = 0 \)) will give the dimensionless force when the trough is at the wall: \( F_t/wd^2 \). For the example problem, with \( H_i/gT^2 = 0.0043 \) and \( H_i/d = 0.50 \),

\[
\frac{F_c}{wd^2} = 0.63; \quad \frac{F_t}{wd^2} = -0.31 \quad \text{(T = 6 s)}
\]

Therefore, assuming a weight per unit volume of 10 kN/m\(^3\) (64.0 lb/ft\(^3\)) for sea water,

\[
F_c = 0.63 \times (10) (3)^2 = 56.7 \text{ kN/m (3,890 lb/ft)} \quad \text{(T = 6 s)}
\]

\[
F_t = -0.31 \times (10) (3)^2 = -27.9 \text{ kN/m (-1,900 lb/ft)} \quad \text{(T = 6 s)}
\]

The values found for \( F_c \) and \( F_t \) do not include the force due to the hydrostatic pressure distribution below the still-water level. For instance, if there is also a water depth of 3 meters (10 feet) on the leeward side of the structure in this example and there is no wave action on the leeward side, then the hydrostatic force on the leeward side exactly balances the hydrostatic force on the side exposed to wave action. Thus, in this case, the values found for \( F_c \) and \( F_t \) are actually the net forces acting on the structure.
If waves act on both sides of the structure, the maximum net horizontal force will occur when the clapotis crest acts against one side when the trough acts against the other. Hence the maximum horizontal force will be \( F_c - F_t \), with \( F_c \) and \( F_t \) determined for the appropriate wave conditions. Assuming for the example problem that the wave action is identical on both sides of the wall, then

\[
F_{net} = 0.63 (10) (3)^2 - (0.31)(10)(3)^2 \\
F_{net} = (0.63 + 0.31) (10) (3)^2 = 84.6 \text{ kN/m (5,800 lb/ft)}
\]

say

\[
F_{net} = 85 \text{ kN/m (T = 6 s)}
\]

Some design problems require calculation of the total force including the hydrostatic contribution; e.g. seawalls. In these cases the total force is found by using equation (7-76). For this example,

\[
F_c \text{ total} = 0.5 (10) (3)^2 + 56.7 = 101.7 \text{ kN/m (7,000 lb/ft)}
\]

\[
F_t \text{ total} = 0.5 (10) (3)^2 + (-27.9) = 17.1 \text{ kN/m (1,200 lb/ft)}
\]

The total force acts against the seaward side of the structure, and the resulting net force will be determined by consideration of static loads (e.g., weight of structure), earth loads (e.g., soil pressure behind a seawall), and any other static or dynamic loading which may occur.

The moment about point A at the bottom of the wall (Fig. 7-89) may be determined from Figure 7-92. The procedures are identical to those given for the dimensionless forces, and again the moment caused by the hydrostatic pressure distribution is not included in the design curves. The upper family of curves (above \( M/wd^3 = 0 \)) gives the dimensionless wave moment when the crest is at the wall, while the lower family of curves corresponds to the trough at the wall. Continuing the example problem, from Figure 7-92, with

\[
\frac{M}{c} = 0.44; \quad \frac{M}{t} = -0.123 \quad \text{(T = 6 s)}
\]

Therefore,

\[
M_c = 0.44 (10) (3)^3 = 118.8 \frac{\text{kN-m}}{\text{m}} (26,700 \frac{\text{lb-ft}}{\text{ft}}) \quad \text{(T = 6 s)}
\]

\[
M_t = -0.123 (10) (3)^3 = -33.2 \frac{\text{kN-m}}{\text{m}} (-7,500 \frac{\text{lb-ft}}{\text{ft}})
\]

7-172
Mc and Mt, given above, are the total moments acting, when there is still water of depth 3 meters (10 feet) on the leeward side of the structure. The maximum moment at which there is wave action on the leeward side of the structure will be $M_c - M_t$ with $M_c$ and $M_t$ evaluated for the appropriate wave conditions prevail on both sides of the structure.

\[
M_{net} = [0.44 - (-0.123)] (10)(3)^3 = 152.0 \text{kN-m/m} \times (34,200 \text{ lb-ft}) \quad (T = 6 \text{ s})
\]

The combined moment due to both hydrostatic and wave loading is found using equation (7-77). For this example,

\[
M_{c total} = \frac{10(3)^3}{6} + 118.8 = 163.8 \text{kN-m/m} \times (36,800 \text{ lb-ft}) \quad (T = 6 \text{ s})
\]

\[
M_{t total} = \frac{10(3)^3}{6} + (-33.2) = 11.8 \text{kN-m/m} \times (2,650 \text{ lb-ft})
\]

Figures 7-93, 7-94, and 7-95 are used in a similar manner to determine forces and moments on a structure which has a reflection coefficient of $\chi = 0.9$.

Wall of Low Height. It is often not economically feasible to design a structure to provide a non-overtopping condition by the design wave. Consequently, it is necessary to evaluate the force on a structure where the crest of the design clapotis is above the top of the wall, as shown in Figure 7-96. When the overtopping is not too severe, the majority of the incident wave will be reflected and the resulting pressure distribution is as shown in Figure 7-96, with the pressure on the wall being the same as in the non-overtopped case. This truncated distribution results in a force $F'$ which is proportional to $F$, the total force that would act against the wall if it extended up to the crest of the clapotis (the force determined from Figures 7-91 or 7-94). The relationship between $F'$ and $F$ is given by

\[
F' = r_f F \quad (7-78)
\]

where $r_f$ is a force reduction factor given by

\[
r_f = \begin{cases} 
\frac{b}{y} \left( 2 - \frac{b}{y} \right) & \text{when } 0.50 < \frac{b}{y} < 1.0 \\
1.0 & \text{when } \frac{b}{y} \geq 1.0
\end{cases}
\quad (7-79)
\]

where $b$ and $y$ are defined in Figure 7-96. The relationship between $r_f$ and $b/y$ is shown in Figure 7-97.
Similarly, the reduced moment about point A is given by

\[ M' = r_m M \]  \hspace{1cm} (7-80)

where the moment reduction factor \( r_m \) is given by

\[
\begin{align*}
    r_m &= \left( \frac{b}{y} \right)^2 \left( 3 - 2 \frac{b}{y} \right) \quad \text{when} \quad 0.50 < \frac{b}{y} < 1.0 \\
    r_m &= 1.0 \quad \text{when} \quad \frac{b}{y} \geq 1.0
\end{align*}
\]  \hspace{1cm} (7-81)

The relationship between \( r_m \) and \( b/y \) is also shown in Figure 7-97. Equations (7-78) through (7-81) are valid when either the wave crest or wave trough is at the structure, provided the correct value of \( y \) is used.

*************** EXAMPLE PROBLEM 32 ***************

**GIVEN:**

(a) Wall height \( b = 4.5 \text{ m} \) (14.8 ft).

(b) Incident wave height \( H_i = 1.5 \text{ m} \) (4.9 ft).

(c) Depth at structure toe \( d = 3 \text{ m} \) (9.8 ft).

(d) Wave period \( T = 6 \text{ s} \) (minimum) or 10 s (maximum).

**FIND:** The reduced wave force and moment on the given vertical wall.
Figure 7-97. Force and moment reduction factors.

\[ r_f = \frac{b}{y}(2 - \frac{b}{y}) \]
\[ r_m = \frac{b}{y}(3 - \frac{b}{y}) \]

where
\[ F = r_f \cdot F \]
\[ M' = r_m \cdot M \]

\[ y = d + h_o \pm \frac{(1+x)h_i}{2} \rightarrow \text{Crest} \]
SOLUTION: From Example Problem 31,

\[ y_c = 5.50 \text{ m (18.1 ft)} \quad (T = 6 \text{ s}) \]

\[ u_t = 2.50 \text{ m (8.2 ft)} \]

Compute \( b/y \) for each case

\[ \frac{b}{y_c} = \frac{4.5}{5.5} = 0.818 \quad (T = 6 \text{ s}) \]

\[ \frac{b}{y_b} = \frac{4.5}{2.5} = 1.80 > 1.0 \]

Entering Figure 7-97 with the computed value of \( b/y \), determine the values of \( r_f \) and \( r_m \) from the appropriate curve. For the wave with \( T = 6 \text{ s} \),

\[ \frac{b}{y_c} = 0.818 \]

therefore,

\[ r_f = 0.968 \]

\[ r_m = 0.912 \]

and

\[ \frac{b}{y_b} > 1.0 \]

therefore,

\[ r_f = 1.0 \]

\[ r_m = 1.0 \]

Reduced forces and moments may be calculated from equations (7-78) and (7-80) using the values of \( F \) and \( M \) found in the example problem of the previous section; for \( T = 6 \text{ s} \).

\[ F'_c = 0.968 \times 101.7 = 98.5 \text{ kN/m (6,750 lb/ft)} \]

\[ M'_c = 0.912 \times 163.8 = 149.4 \text{ kN-m (33,590 lb-ft/ft)} \]

\[ F'_t = 1.0 \times 17.1 = 17.1 \text{ kN/m (1,200 lb/ft)} \]

\[ M'_t = 1.0 \times 11.8 = 11.8 \text{ kN-m/m (2,650 lb-ft/ft)} \]
These values include the force and moment due to the hydrostatic component of the loading.

Again assuming that the wave action on both sides of the structure is identical, so that the maximum net horizontal force and maximum overturning moment occurs when a clapotis crest is on one side of the structure and a trough is on the other side

\[
F'_{\text{net}} = F'_c - F'_t = 98.5 - 17.1 = 81.4 \text{ kN/m}
\]

say \[F'_{\text{net}} = 82 \text{ kN/m (5,620 lb/ft)} \] (T = 6 s)

and

\[
M'_{\text{net}} = M'_c - M'_t = 149.4 - 11.8 = 137.6 \text{ kN-m/m}
\]

say \[M'_{\text{net}} = 138 \text{ kN-m/m (31,000 lb-ft/ft)} \] (T = 6 s)

A similar analysis for the 10-second wave gives,

\[
F'_{\text{net}} = 85.2 \text{ kN/m (5,840 lb/ft)} \]

\[M'_{\text{net}} = 139 \text{ kN/m (31,250 lb-ft/ft)} \] (T = 10 s)

e. Wall on Rubble Foundation. Forces acting on a vertical wall built on a rubble foundation are shown in Figure 7-98 and may be computed in a manner similar to computing the forces acting on a low wall if the complements of the force and moment reduction factors are used. As shown in Figure 7-98, the value of \(b\) which is used for computing \(b/y\) is the height of the rubble base and not the height of the wall above the foundation. The equation relating the reduced force \(F''\) against the wall on a rubble foundation with the force \(F\) which would act against a wall extending the entire depth is

\[
F'' = \left(1 - r_f\right) F \tag{7-82}
\]

The equation relating the moments is,

\[
M'' = \left(1 - r_m\right) M \tag{7-83}
\]
where $M''_A$ is the moment about the bottom (point A on Fig. 7-98). Usually, the moment desired is that about point B, which may be found from

$$M''_B = \left(1 - r_m\right) M - b \left(1 - r_f\right) F$$

or

$$M''_B = M''_A - b F''$$

The values of $(1 - r_m)$ and $(1 - r_f)$ may be obtained directly from Figure 7-97.

*************** EXAMPLE PROBLEM 33 ***************

**GIVEN:**

(a) A smooth-faced vertical wall on a rubble base.

(b) Height of rubble foundation, $b = 2.7$ m (9 ft).

(c) Incident wave height $H_i = 1.5$ m (5 ft).

(d) Design depth at the structure $d = 3$ m (10 ft).

(e) Wave period $T = 6$ s (minimum) or 10 s (maximum).

**FIND:** The force and overturning moment on the given wall on a rubble foundation.

**SOLUTION:** For this example problem Figures 7-90 through 7-92 are used to
evaluate \( h_o \), \( F \), and \( M \), even though a rubble base will reduce the wave reflection coefficient of a structure by dissipating some incident wave energy. The values of \( h_o \), \( F \), and \( M \) used in this example were determined in Example Problem 31.

\[
y_c = 5.5 \text{ m} \\
y_t = 2.5 \text{ m}
\]

Compute \( b/y \) for each case, remembering that \( b \) now represents the height of the foundation.

\[
\frac{b}{y_c} = \frac{2.7}{5.5} = 0.491 \\
\frac{b}{y_t} = \frac{2.7}{2.5} = 1.08 > 1.0
\]

Enter Figure 7-97 with the computed values of \( b/y \), and determine corresponding values of \((1 - r_f)\) and \((1 - r_m)\). For the 6-second wave,

\[
\frac{b}{y_c} = 0.491; (1 - r_f) = 0.26; (1 - r_m) = 0.52
\]

and

\[
\frac{b}{y_t} > 1.0; (1 - r_f) = 0.0; (1 - r_m) = 0.0
\]

From equation (7-82),

\[
F''_c = 0.26 (101.7) = 26.5 \text{ kN/m} (1,820 \text{ lb/ft}) \\
F''_t = 0.0 (17.1) = 0 \text{ kN/m}
\]

For the 10-second wave, a similar analysis gives

\[
F''_c = 30.8 \text{ kN/m} (2,100 \text{ lb/ft}) \\
F''_t \approx 0 \text{ kN/m}
\]

The overturning moments about point \( A \) are, from equation (7-83)

\[
(M''_A)_c = 0.52 (163.8) = 85.2 \text{ kN-m/m} (19,200 \text{ lb-ft/ft}) \\
(M''_A)_t = 0.0 (11.8) = 0 \text{ kN-m/m}
\]
and for the 10-second wave,

\[
\frac{(M'')_{Bc}}{A} = 95.9 \frac{\text{kN-m}}{m} (21,600 \frac{\text{lb-ft}}{ft})
\]

\[
\frac{(M'')_{Bt}}{A} = 0 \frac{\text{kN-m}}{m}
\]

The overturning moments about point \( B \) are obtained from equation (7-84) thus

\[
(M''_B)_c = 85.2 - 2.7 (26.5) = 13.7 \frac{\text{kN-m}}{m} (3,080 \frac{\text{lb-ft}}{ft})
\]

\[
(M''_B)_t = 0 \text{kN-m/m}
\]

and for the 10-second wave,

\[
(M''_B)_c = 12.7 \frac{\text{kN-m}}{m} (2,850 \frac{\text{lb-ft}}{ft})
\]

\[
(M''_B)_t = 0 \text{kN-m/m}
\]

As in Examples Problems 31 and 32, various combinations of appropriate wave conditions for the two sides of the structure can be assumed and resulting moments and forces computed.

3. **Breaking Wave Forces on Vertical Walls.**

Waves breaking directly against vertical-face structures exert high, short duration, dynamic pressures that act near the region where the wave crests hit the structure. These impact or shock pressures have been studied in the laboratory by Bagnold (1939), Denny (1951), Ross (1955), Carr (1954), Leendertse (1961), Nagai (1961a), Kamel (1968), Weggel (1968), and Weggel and Maxwell (1970a and b). Some measurements on full-scale breakwaters have been made by deRouville et al., (1938) and by Nuraki (1966). Additional references and discussion of breaking wave pressures are given by Silvester (1974). Wave tank experiments by Bagnold (1939) led to an explanation of the phenomenon. Bagnold found that impact pressures occur at the instant that the vertical front face of a breaking wave hits the wall and only when a plunging wave entraps a cushion of air against the wall. Because of this critical dependence on wave geometry, high impact pressures are infrequent against prototype structures; however, the possibility of high impact pressures must be recognized and considered in design. Since the high impact pressures are short (on the order of hundredths of a second), their importance in the design of breakwaters against sliding or overturning is questionable; however, lower dynamic forces which last longer are important.
Minikin Method: Breaking Wave Forces. Minikin (1955, 1963) developed a design procedure based on observations of full-scale breakwaters and the results of Bagnold’s study. Minikin’s method can give wave forces that are extremely high, as much as 15 to 18 times those calculated for nonbreaking waves. Therefore, the following procedures should be used with caution and only until a more accurate method of calculation is found.

The maximum pressure assumed to act at the SWL is given by

\[ p_m = 101 \, w \frac{H_b}{D} \frac{d_s}{D} \left( D + d_s \right) \]  \hspace{1cm} (7-85)

where \( p_m \) is the maximum dynamic pressure, \( H_b \) is the breaker height, \( d_s \) is the depth at the toe of the wall, \( D \) is the depth one wavelength in front of the wall, and \( L_D \) is the wavelength in water of depth \( D \). The distribution of dynamic pressure is shown in Figure 7-99. The pressure decreases parabolically from \( p_m \) at the SWL to zero at a distance of \( H_b / 2 \) above and below the SWL. The force represented by the area under the dynamic pressure distribution is

\[ R_m = \frac{p_m H_b D}{3} \]  \hspace{1cm} (7-86)

i.e., the force resulting from dynamic component of pressure and the overturning moment about the toe is

\[ M_m = R_m d_s = \frac{p_m H_b D d_s}{3} \]  \hspace{1cm} (7-87)

i.e., the moment resulting from the dynamic component of pressure. The hydrostatic contribution to the force and overturning moment must be added to the results obtained from equations (7-86) and (7-87) to determine total force and overturning moment.

Figure 7-99. Minikin wave pressure diagram.
The Minikin formula was originally derived for composite breakwaters composed of a concrete superstructure founded on a rubble substructure; strictly, $D$ and $L_D$ in equation (7-85) are the depth and wavelength at the toe of the substructure, and $d_s$ is the depth at the toe of the vertical wall (i.e., the distance from the SWL down to the crest of the rubble substructure). For caisson and other vertical structures where no substructure is present, the formula has been adapted by using the depth at the structure toe as $d_s$, while $D$ and $L_D$ are the depth and wavelength a distance one wavelength seaward of the structure. Consequently, the depth $D$ can be found from

$$D = d_s + L_d$$  \hspace{1cm} (7-88)

where $L_d$ is the wavelength in a depth equal to $d_s$, and $m$ is the nearshore slope. The forces and moments resulting from the hydrostatic pressure must be added to the dynamic force and moment computed above. The triangular hydrostatic pressure distribution is shown in Figure 7-99; the pressure is zero at the breaker crest (taken at $H_b/2$ above the SWL), and increases linearly to $w(d_s + H_b/2)$ at the toe of the wall. The total breaking wave force on a wall per unit wall length is

$$R_t = R_m + \frac{w\left(d_s + \frac{H_b}{2}\right)^2}{2} = R_m + R_e$$  \hspace{1cm} (7-89)

where $R_e$ is the hydrostatic component of breaking wave on a wall, and the total moment about the toe is

$$M_t = M_m + \frac{w\left(d_s + \frac{H_b}{2}\right)^3}{6} = M_m + M_e$$  \hspace{1cm} (7-90)

where $M_e$ is the hydrostatic moment.

Calculations to determine the force and moment on a vertical wall are illustrated by the following example.

************** EXAMPLE PROBLEM 34 **************

**GIVEN:** A vertical wall, 4.3 m (14 ft) high is sited in sea water with $d_s = 2.5$ m (8.2 ft). The wall is built on a bottom slope of 1:20 ($m = 0.05$) Reasonable wave periods range from $T = 6$ s to $T = 10$ s.

**FIND:**

(a) The maximum pressure, horizontal force, and overturning moment about the toe of the wall for the given slope.

(b) The maximum pressure, horizontal force, and overturning moment for the 6-second wave if the slope was 1:100.
SOLUTION:

(a) From Example Problem 3, the maximum breaker heights for a design depth of 2.5 m (8.2 ft), a slope of 0.05, and wave periods of 6- and 10-seconds are

\[ H_b = 2.8 \text{ m (9.2 ft)} \quad (T = 6 \text{ s}) \]

\[ H_b = 3.2 \text{ m (10.5 ft)} \quad (T = 10 \text{ s}) \]

The wavelength at the wall in water 2.5 m (8.2 ft) deep can be found with the aid of Table C-1, Appendix C. First calculate the wavelength in deep water \((T = 6 \text{ s})\),

\[ L_o = \frac{gT^2}{2\pi} = 1.56 \times (6)^2 = 56.2 \text{ m (184 ft)} \]

Then

\[ \frac{d}{L_o} = \frac{2.5}{56.2} = 0.04448 \]

and from Table C-1, Appendix C,

\[ \frac{d}{L} = 0.08826 \]

and

\[ L_d = 28.3 \text{ m (92.8 ft)} \]

from equation (7-88)

\[ D = d_s + L_d \quad m = 2.5 + 28.3 (0.05) = 3.9 \text{ m (12.8 ft)} \]

and using Table C-1, as above,

\[ \frac{D}{L_o} = 0.06940; \quad \frac{D}{L_d} = 0.1134 \]

hence

\[ L_D = \frac{D}{\frac{D}{L_D}} = \frac{3.9}{0.1134} = 34.4 \text{ m} \]

say

\[ L_D = 35 \text{ m (115 ft)} \]
Equation (7-85) can now be used to find $p_m$.

$$p_m = 101 \frac{w}{L_D} \frac{d g}{D} \left( D + d_g \right)$$

$$p_m = 101 \cdot (10) \frac{2.8}{35} \frac{2.5}{3.9} (3.9 + 2.5)$$

$$= 331 \text{kN/m}^2 (6,913 \text{lb/ft}^2) \quad (T = 6 \text{s})$$

A similar analysis for the 10-second wave gives,

$$p_m = 182 \text{kN/m}^2 (3,801 \text{lb/ft}^2) \quad (T = 10 \text{s})$$

The above values can be obtained more rapidly by using Figure 7-100, a graphical representation of the above procedure. To use the figure, calculate for the 6-second wave,

$$\frac{d g}{gT^2} = \frac{2.5}{9.81 (6)^2} = 0.0071$$

Enter Figure 7-100 with the calculated value of $d_g/gT^2$, using the curve for $m = 0.05$, and read the value of $p_m/wH_b$.

$$\frac{p_m}{wH_b} = 12.0$$

Using the calculated values of $H_b$

$$p_m = 12.0wH_b = 12.0 (10) (2.8) = 336 \text{kN/m}^2 (7,017 \text{lb/ft}^2)$$

For the 10-second wave,

$$p_m = 5.5 \cdot wH_b = 5.5 (10) (3.2) = 176 \text{kN/m}^2 (3,676 \text{lb/ft}^2) \quad (T = 10 \text{s})$$

The force can be evaluated from equation (7-86) thusly:

$$R_m = \frac{p_m H_b}{3} = \frac{331 (2.8)}{3} = 309 \text{ kN/m (21,164 lb/ft)} \quad (T = 6 \text{s})$$

and

$$R_m = 194 \text{ kN/m (13,287 lb/ft)} \quad (T = 10 \text{s})$$

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Figure 7-100. Dimensionless Minikin wave pressure and force.
The overturning moments are given by equation (7-87) as

\[ M_m = R_m d_g = 309 \times (2.5) = 772 \frac{\text{kN-m}}{\text{m}} \left(173,561 \frac{\text{ft-lb}}{\text{ft}}\right) \quad (T = 6 \text{ s}) \]

and

\[ M_m = 485 \frac{\text{kN/m}}{\text{m}} \left(109,038 \frac{\text{ft-lb}}{\text{ft}}\right) \quad (T = 10 \text{ s}) \]

For the example, the total forces, including the hydrostatic force from equations (7-89) and (7-90),

\[ R_t = R_m + R_s \]

\[ R_t = 309 + \frac{10 \left(2.5 + \frac{2.8}{2}\right)^2}{2} = 309 + 76 = 385 \frac{\text{kN/m}}{\text{m}} \left(26,382 \frac{\text{lb/ft}}{\text{ft}}\right) \quad (T = 6 \text{ s}) \]

\[ R_t = 278 \frac{\text{kN/m}}{\text{m}} \left(19,041 \frac{\text{lb/ft}}{\text{ft}}\right) \quad (T = 10 \text{ s}) \]

Then

\[ M_t = M_m + M_s \]

\[ M_t = 772 + \frac{10 \left(2.5 + \frac{2.8}{2}\right)^3}{6} = 772 + 99 \quad (T = 6 \text{ s}) \]

\[ M_t = 871 \frac{\text{kN/m}}{\text{m}} \left(195,818 \frac{\text{ft-lb}}{\text{ft}}\right) \quad (T = 10 \text{ s}) \]

and

\[ M_t = 600 \frac{\text{kN-m}}{\text{m}} \left(134,892 \frac{\text{ft-lb}}{\text{ft}}\right) \]

(b) If the nearshore slope is 1:100 \((m = 0.01)\), the maximum breaker heights must be recomputed using the procedure given in Section I.2,b. For a 6-second wave on a 0.01 slope the results of an analysis similar to the preceding gives

\[ H_b = 2.1 \text{ m} \quad (d_b = 2.6 \text{ m} \ (7.9 \text{ ft}) > d_s) \]

\[ p_m = 337 \frac{\text{kN/m}^2}{\text{m}^2} \left(7,035 \frac{\text{lb/ft}^2}{\text{ft}^2}\right) \quad (T = 6 \text{ s}) \]
and

\[ R_m = 236 \text{ kN/m (16,164 lb/ft)} \]

The resulting maximum pressure is about the same as for the wall on a 1:20 sloping beach. \((p_m = 336 \text{ kN/m})\); however, the dynamic force is less against the wall on a 1:100 slope than against the wall on a 1:20 slope, because the maximum possible breaker height reaching the wall is lower on a flatter slope.

b. **Wall On a Rubble Foundation.** The dynamic component of breaking wave force on a vertical wall built on a rubble substructure can be estimated with either equation (7-85) or Figure 7-101. The procedure for calculating forces and moments is similar to that outlined in the Example Problem 34, except that the ratio \(d_s/D\) is used instead of the nearshore slope when using Figure 7-101. Minikin's equation was originally derived for breakwaters of this type. For expensive structures, hydraulic models should be used to evaluate forces.

c. **Wall of Low Height.** When the top of a structure is lower than the crest of the design breaker, the dynamic and hydrostatic components of wave force and overturning moment can be corrected by using Figures 7-102 and 7-103. Figure 7-102 is a Minikin force reduction factor to be applied to the dynamic component of the breaking wave force equation

\[ R'_m = r_m R_m \quad (7-91) \]

Figure 7-103 gives a moment reduction factor \(a\) for use in the equation

\[ M'_m = d_s R_m - (d_s + a) (1 - r_m) R_m \quad (7-92) \]

or

\[ M'_m = R_m [r_m (d_m + a) - a] \quad (7-93) \]

* * * * * * * * * * * * * EXAMPLE PROBLEM 35 * * * * * * * * * * * * * *

**GIVEN:**

(a) A vertical wall 3 m (10 ft) high in a water depth of \(d_s = 2.5 \text{ m (8.2 ft)}\) on a nearshore slope of 1:20 \((m = 0.05)\);

(b) Design wave periods of \(T = 6 \text{ s}\) and \(= 10 \text{ s}\).

**FIND:** The reduced force and overturning moment because of the reduced wall height.

**SOLUTION:** Calculations of the breaker heights, unreduced forces, and moments are given in preceding example problems. From the preceding problems,

\[ H_b = 2.8 \text{ m (9.2 ft)} \quad (d_b = 3.0 \text{ m} > d_s) \]
Figure 7-101. Dimensionless Minikin wave pressure and force.
Figure 7-102. Minikin force reduction factor.
Figure 7-103. Minikin moment reduction for low wall.

\[ M'_m = d_s R_m - (d_s + a) (1 - r_m) R_m \]
\[ R_m = 309 \text{kN/m (21,164 lb/ft)} \]

\[ M_m = 772 \frac{\text{kN-m}}{m} \left( 173,561 \frac{\text{ft-lb}}{\text{ft}} \right) \quad (T = 6 \text{s}) \]

and

\[ H_b = 3.2 \text{ m (10.5 ft)} \quad (d_b = 3.0 \text{ m} > d_s) \]

\[ R_m = 194 \text{kN/m (13,287 lb/ft)} \]

\[ M_m = 485 \text{kN-m/m (109,038 ft-lb/ft)} \quad (T = 10 \text{s}) \]

For the breaker with a period of 6 seconds, the height of the breaker crest above the bottom is

\[ \left( d_s + \frac{H_b}{2} \right) = \left( 2.5 + \frac{2.8}{2} \right) = 3.9 \text{ m (12.8 ft)} \]

The value of \( b' \) as defined in Figure 7-102 is 1.9 m (6.2 ft) (i.e., the breaker height \( H_b \) minus the height obtained by subtracting the wall crest elevation from the breaker crest elevation). Calculate

\[ \frac{b'}{H_b} = \frac{1.9}{2.8} = 0.679 \quad (T = 6 \text{s}) \]

From Figure 7-102,

\[ r_m = 0.83 \]

displacement from equation (7-91),

\[ R_m' = r_m \ R_m = 0.83 \ (309) = 256 \text{kN/m (17,540 lb/ft)} \quad (T = 6 \text{s}) \]

From Figure 7-103, entering with \( b/H_b' = 0.679 \),

\[ \frac{2a}{H_b} = 0.57 \]

hence

\[ a = \frac{0.57 \times 2.8}{2} = 0.80 \text{ m} \]

and from equation (7-93)

\[ M_m' = R_m \ [r_m \ (d_s + a) - a] = 309 \ [0.83 \ (2.5+0.80) -0.80] \]

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\[ M'_{m} = 309 [1.94] = 600 \text{ kN-m/m (134,900 ft-lb/ft)} \quad (T = 6 \text{ s}) \]

A similar analysis for the maximum breaker with a 10-second period gives

\[ r_m = 0.79 \]
\[ a = 0.86 \text{ m (2.82 ft)} \]
\[ R'_{m} = 153 \text{ kN/m (10,484 lb/ft)} \]

\[ M_m = 348 \frac{\text{kN-m}}{\text{m}} \left( 78,237 \frac{\text{lb-ft}}{\text{ft}} \right) \quad (T = 10 \text{ s}) \]

The hydrostatic part of the force and moment can be computed from the hydrostatic pressure distribution shown in Figure 7-99 by assuming the hydrostatic pressure to be zero at \( H_b/2 \) above SWL and taking only that portion of the area under the pressure distribution which is below the crest of the wall.

4. **Broken Waves.**

Shore structures may be located so that even under severe storm and tide conditions waves will break before striking the structure. No studies have yet been made to relate forces of broken waves to various wave parameters, and it is necessary to make simplifying assumptions about the waves to estimate design forces. If more accurate force estimates are required, model tests are necessary.

It is assumed that, immediately after breaking, the water mass in a wave moves forward with the velocity of propagation attained before breaking; that is, upon breaking, the water particle motion changes from oscillatory to translatory motion. This turbulent mass of water then moves up to and over the stillwater line dividing the area shoreward of the breakers into two parts, seaward and landward of the stillwater line. For a conservative estimate of wave force, it is assumed that neither wave height nor wave velocity decreases from the breaking point to the stillwater line and that after passing the stillwater line the wave will run up roughly twice its height at breaking, with both velocity and height decreasing to zero at this point. Wave runup can be estimated more accurately from the procedure outlined in Section 1, Wave Runup.

Model tests have shown that, for waves breaking at a shore, approximately 78 percent of the breaking wave height \( H_b \) is above the stillwater level (Wiegel, 1964).

a. **Wall Seaward of Stillwater Line.** Walls located seaward of the stillwater line are subjected to wave pressures that are partly dynamic and partly hydrostatic (see Figure 7-104).

Using the approximate relationship \( C = \sqrt{g d_b} \) for the velocity of wave propagation, \( C \) where \( g \) is the acceleration of gravity and \( d_b \) is the
breaking wave depth, wave pressures on a wall may be approximated in the following manner:

The dynamic part of the pressure will be

\[ p_m = \frac{wc^2}{2g} = \frac{wd_b}{2} \quad (7-94) \]

Figure 7-104. Wave pressures from broken waves: wall seaward of still-water line.

where \( w \) is the unit weight of water. If the dynamic pressure is uniformly distributed from the still-water level to a height \( h_c \) above SWL, where \( h_c \) is given as

\[ h_c = 0.78H_b \quad (7-95) \]

then the dynamic component of the wave force is given as

\[ R_m = p_m h_c = \frac{wd_b h_c}{2} \quad (7-96) \]

and the overturning moment caused by the dynamic force as

\[ M_m = R_m \left( d_s + \frac{h_s}{2} \right) \quad (7-97) \]

where \( d_s \) is the depth at the structure.

The hydrostatic component will vary from zero at a height \( h_c \) above SWL to a maximum \( p_S \) at the wall base. This maximum will be given as,

\[ p_S = w \left( d_s + h_s \right) \quad (7-98) \]
The hydrostatic force component will therefore be

$$R_g = \frac{w (d_g + h_c)^2}{2}$$  \hspace{1cm} (7-99)$$

and the overturning moment will be,

$$M_g = R_g \left( \frac{d_g + h_c}{3} \right) = \frac{w (d_g + h_c)^3}{6}$$  \hspace{1cm} (7-100)$$

The total force on the wall is the sum of the dynamic and hydrostatic components; therefore,

$$R_t = R_m + R_s$$  \hspace{1cm} (7-101)$$

and

$$M_t = M_m + M_s$$  \hspace{1cm} (7-102)$$

b. Wall Shoreward of Still-water Line. For walls landward of the still-water line as shown in Figure 7-105, the velocity $v'$ of the water mass at the structure at any location between the SWL and the point of maximum wave runup may be approximated by,

$$v' = C \left( 1 - \frac{x_1}{x_2} \right) = \sqrt{gd_b} \left( 1 - \frac{x_1}{x_2} \right)$$  \hspace{1cm} (7-103)$$

and the wave height $h'$ above the ground surface by

$$h' = h_c \left( 1 - \frac{x_1}{x_2} \right)$$  \hspace{1cm} (7-104)$$

where

$$x_1 = \text{distance from the still-water line to the structure}$$

$$x_2 = \text{distance from the still-water line to the limit of wave uprush; i.e., } x_2 = 2H_b \cot \beta = 2H_b / m \text{ (note: the actual wave runup as found from the method outlined in Section II,1 could be substituted for the value } 2H_b)$$

$$\beta = \text{the angle of beach slope}$$

$$m = \tan \beta$$

An analysis similar to that for structures located seaward of the still-water line gives for the dynamic pressure

$$p_m = \frac{wv'^2}{2g} = \frac{wd_b}{2} \left( 1 - \frac{x_1}{x_2} \right)^2$$  \hspace{1cm} (7-105)$$

7-194
The dynamic pressure is assumed to act uniformly over the broken wave height at the structure toe $h'$, hence the dynamic component of force is given by

$$R_m = p_m h' = \frac{w d_b h}{2} \left( 1 - \frac{x_1}{x_2} \right)^3$$  \hspace{1cm} (7-106)

and the overturning moment by

$$M_m = \frac{R_m h'}{2} = \frac{w d_b h^2}{4} \left( 1 - \frac{x_1}{x_2} \right)^4$$  \hspace{1cm} (7-107)

The hydrostatic force component is given by

$$R_h = \frac{w h'^2}{2} = \frac{w h^2}{2} \left( 1 - \frac{x_1}{x_2} \right)^2$$  \hspace{1cm} (7-108)

and the moment resulting from the hydrostatic force by
The total forces and moments are the sums of the dynamic and hydrostatic components; therefore, as before,

\[ R_t = R_m + R_s \tag{7-110} \]

and

\[ M_t = M_m + M_s \tag{7-111} \]

The pressures, forces, and moments computed by the above procedure will be approximations, since the assumed wave behavior is simplified. Where structures are located landward of the still-water line the preceding equations will not be exact, since the runup criterion was assumed to be a fixed fraction of the breaker height. However, the assumptions should result in a high estimate of the forces and moments.

********** EXAMPLE PROBLEM 36 **********

**GIVEN:** The elevation at the toe of a vertical wall is 0.6 m (2 ft) above the mean lower low water (MLLW) datum. Mean higher high water (MHHW) is 1.3 m (4.3 ft) above MLLW, and the beach slope is 1:20. Breaker height is \( H_b = 3.0 \) m (9.8 ft), and wave period is \( T = 6 \) s.

**FIND:**

(a) The total force and moment if the SWL is at MHHW; i.e., if the wall is seaward of still-water line.

(b) The total force and moment if the SWL is at MLLW; i.e., if the wall is landward of still-water line.

**SOLUTION:**

(a) The breaking depth \( d_b \) can be found from Figure 7-2. Calculate,

\[ \frac{H_b}{gT^2} = \frac{3.0}{9.8 \times (6)^2} = 0.0085 \]

and the beach slope,

\[ m = \tan \beta = 0.05 \]

Enter Figure 7-2 with \( H_b / gT^2 = 0.0085 \) and, using the curve for \( m = 0.05 \), read

\[ \frac{d_b}{H_b} = 1.10 \]
Therefore,
\[ d_b = 1.10 H_b = 1.10 \times 3.0 = 3.3 \text{ m (10.8 ft)} \]

From equation (7-95)
\[ h_c = 0.78 H_b = 0.78 \times 3.0 = 2.3 \text{ m (7.7 ft)} \]

The dynamic force component from equation (7-96) is
\[ R_m = \frac{wd_b h_c}{2} = \frac{10,047 \times (3.3) \times (2.3)}{2} = 38.1 \text{ kN/m (2,610 lb/ft)} \]

and the moment from equation (7-97) is
\[ M_m = R_m \left( d_c + \frac{h_c}{2} \right) = 38.1 \left( 0.7 + \frac{2.3}{2} \right) = 70.5 \frac{\text{kN-m}}{\text{m}} \left( 15,900 \frac{\text{ft-lb}}{\text{ft}} \right) \]

where \( d_c = 0.7 \text{ m} \) is the depth at the toe of the wall when the SWL is at MHHW. The hydrostatic force and moment are given by equations (7-99) and (7-100):
\[ R_g = \frac{w \left( d_g + h_c \right)^2}{2} = \frac{10,047 \times (0.7 + 2.3)^2}{2} = 45.2 \text{ kN/m (3,100 lb/ft)} \]
\[ M_g = R_g \left( d_g + h_c \right) = 45,212 \left( 0.7 + \frac{2.3}{3} \right) = 45.2 \frac{\text{kN-m}}{\text{m}} \left( 10,200 \frac{\text{ft-lb}}{\text{ft}} \right) \]

The total force and moment are therefore,
\[ R_t = R_m + R_g = 38.1 + 45.2 = 83.3 \text{ kN/m (5,710 lb/ft)} \]
\[ M_t = M_m + M_g = 70.5 + 45.2 = 115.7 \frac{\text{kN-m}}{\text{m}} \left( 26,000 \frac{\text{ft-lb}}{\text{ft}} \right) \]

(b) When the SWL is at MLLW, the structure is landward of the still-water line. The distance from the still-water line to the structure \( x_1 \) is given by the difference in elevation between the SWL and the structure toe divided by the beach slope; hence
\[ x_1 = \frac{0.6}{0.05} = 12 \text{ m (39.4 ft)} \]

The limit of wave runup is approximately
\[ x_2 = \frac{2H_b}{m} = \frac{2 \times 3.0}{0.05} = 120 \text{ m} \]

7-197
The dynamic component of force from equation (7-106) is,

\[
R_m = \frac{wd_b h_c}{2} \left(1 - \frac{x_1}{x_2}\right)^3 = 10,047 \frac{(3.3)(2.3)}{2} \left(1 - \frac{12}{120}\right)^3 = 27.8 \text{ kN/m (1,905 lb/ft)}
\]

and the moment from equation (7-107) is

\[
M_m = \frac{wd_b h_c^2}{4} \left(1 - \frac{x_1}{x_2}\right)^4 = 10,047 \frac{(3.3)(2.3)^2}{4} \left(1 - \frac{12}{120}\right)^4 = 28.8 \text{ kN-m m (6,500 ft-lb)}
\]

The hydrostatic force and moment from equations (7-108) and (7-109) are,

\[
R_s = \frac{wh^2}{2} \left(1 - \frac{x_1}{x_2}\right)^2 = 10,047 \frac{(2.3)^2}{2} \left(1 - \frac{12}{120}\right)^2 = 21.5 \text{ kN/m (1,475 lb/ft)}
\]

and

\[
M_s = \frac{wh^3}{6} \left(1 - \frac{x_1}{x_2}\right)^3 = 10,047 \frac{(2.3)^3}{6} \left(1 - \frac{12}{120}\right)^3 = 14.9 \text{ kN-m m (3,400 ft-lb)}
\]

Total force and moment are

\[
R_t = R_m + R_s = 27.8 + 21.5 = 49.3 \text{ kN/m (3,400 lb/ft)}
\]

\[
M_t = M_m + M_s = 28.8 + 14.9 = 43.7 \text{ kN-m m (9,800 ft-lb)}
\]


When breaking or broken waves strike the vertical face of a structure such as a groin, bulkhead, seawall, or breakwater at an oblique angle, the dynamic component of the pressure or force will be less than for breaking or broken waves that strike perpendicular to the structure face. The force may be reduced by the equation,

\[
R' = R \sin^2 \alpha \quad (7-112)
\]

where \( \alpha \) is the angle between the axis of the structure and the direction of wave advance, \( R' \) is the reduced dynamic component of force, and \( R \) is the dynamic force that would occur if the wave hit perpendicular to the structure. The development of equation (7-112) is given in Figure 7-106. Force reduction by equation (7-112) should be applied only to the dynamic wave-force Component of breaking or broken waves and should not be applied to the
R = Dynamic Force Per Unit Length of Wall if Wall were Perpendicular to Direction of Wave Advance

\[ R_n = \text{Component of } R \text{ Normal to Actual Wall. } R_n - R \sin \alpha \]

W = Length Along Wall Affected by a Unit Length of Wave Crest. \[ W = \frac{1}{\sin \alpha} \]

R' = Dynamic Force Per Unit Length of Wall

\[ R' = \frac{R_n}{W} = \frac{R \sin \alpha}{1/\sin \alpha} = R \sin^2 \alpha \]

R' = R \sin^2 \alpha

Figure 7-106. Effect of angle of wave approach: plan view.
hydrostatic component. The reduction is not applicable to rubble structures. The maximum force does not act along the entire length of a wall simultaneously; consequently, the average force per unit length of wall will be lower.

6. Effect of a Nonvertical Wall.

Formulas previously presented for breaking and broken wave forces may be used for structures with nearly vertical faces.

If the face is sloped backward as in Figure 7-107a, the horizontal component of the dynamic force due to waves breaking either on or seaward of the wall should be reduced to

\[ R'' = R' \sin^2 \theta \]  

(7-113)

where \( \theta \) is defined in Figure 7-107. The vertical component of the dynamic wave force may be neglected in stability computations. For design calculations, forces on stepped structures as in Figure 7-107b may be computed as if the face were vertical, since the dynamic pressure is about the same as computed for vertical walls. Curved nonreentrant face structures (Fig. 7-107c) and reentrant curved face walls (Fig. 7-107d) may also be considered as vertical.

Figure 7-107. Wall shapes.
EXAMPLE PROBLEM 37

**GIVEN:** A structure in water, $d_s = 2.3$ m (7.5 ft), on a 1:20 nearshore slope, is subjected to breaking waves, $H_b = 2.6$ m (8.4 ft) and period $T = 6$ s. The angle of wave approach is, $\alpha = 80^\circ$, and the wall has a shoreward sloping face of 10 (vertical) on 1 (horizontal).

**FIND:**

(a) The reduced total horizontal wave force.

(b) The reduced total overturning moment about the toe (Note: neglect the vertical component of the hydrostatic force).

**SOLUTION:** From the methods used in Example Problems 34 and 36 for the given wave conditions, compute

\[ R_m = 250 \text{ kN/m (17,100 lb/ft)} \]

\[ M_m = 575 \frac{\text{kN-m}}{m} (129,300 \frac{\text{ft-lb}}{\text{ft}}) \]

\[ R_s = 65 \text{ kN/m (4,450 lb/ft)} \]

and

\[ M_\theta = 78 \frac{\text{kN-m}}{m} (17,500 \frac{\text{ft-lb}}{\text{ft}}) \]

Applying the reduction of equation (7-112) for the angle of wave approach, with $R_m = R$

\[ R' = R_m \sin^2 \alpha = 250 (\sin 80^\circ)^2 \]

\[ R' = 250 (0.985)^2 = 243 \text{ kN/m (16,700 lb/ft)} \]

Similarly,

\[ M' = M_m \sin^2 \alpha = 575 (\sin 80^\circ)^2 \]

\[ M' = 575 (0.985)^2 = 558 \frac{\text{kN-m}}{m} (125,500 \frac{\text{ft-lb}}{\text{ft}}) \]

Applying the reduction for a nonvertical wall, the angle the face of the wall makes with the horizontal is

\[ \theta = \arctan (10) \approx 84^\circ \]

Applying equation (7-113),

\[ R'' = R' \sin^2 \theta = 243 (\sin 84^\circ)^2 \]
R" = 243 (0.995)^2 = 241 kN/m (16,500 lb/ft)

Similarly, for the moment

M" = M' sin^2 \theta = 558 (\sin 84^\circ)^2

M" = 558 (0.995)^2 = 553 \frac{kN-m}{m} (124,200 \frac{ft-lb}{ft})

The total force and overturning moment are given by the sums of the reduced dynamic components and the unreduced hydrostatic components. Therefore,

R_t = 241 + 65 = 306 kN/m (21,000 lb/ft)

M_L = 553 + 78 = 631 \frac{kN-m}{m} (141,900 \frac{ft-lb}{ft})

7. Stability of Rubble Structures.

a. General. A rubble structure is composed of several layers of random-shaped and random-placed stones, protected with a cover layer of selected armor units of either quarystone or specially shaped concrete units. Armor units in the cover layer may be placed in an orderly manner to obtain good wedging or interlocking action between individual units, or they may be placed at random. Present technology does not provide guidance to determine the forces required to displace individual armor units from the cover layer. Armor units may be displaced either over a large area of the cover layer, sliding down the slope en masse, or individual armor units may be lifted and rolled either up or down the slope. Empirical methods have been developed that, if used with care, will give a satisfactory determination of the stability characteristics of these structures when under attack by storm waves.

A series of basic decisions must be made in designing a rubble structure. Those decisions are discussed in succeeding sections.

b. Design Factors. A primary factor influencing wave conditions at a structure site is the bathymetry in the general vicinity of the structure. Depths will partly determine whether a structure is subjected to breaking, nonbreaking, or broken waves for a particular design wave condition (see Section I, WAVE CHARACTERISTICS).

Variation in water depth along the structure axis must also be considered as it affects wave conditions, being more critical where breaking waves occur than where the depth may allow only nonbreaking waves or waves that overtop the structure.

When waves impinge on rubble structures, they do the following:

(a) Break completely, projecting a jet of water roughly perpendicular to the slope.
(b) Partially break with a poorly defined jet.

(c) Establish an oscillatory motion of the water particles up or down the structure slope, similar to the motion of a clapotis at a vertical wall.

The design wave height for a flexible rubble structure should usually be the average of the highest 10 percent of all waves, $H_{10}$, as discussed in Section 1.2. Damage from waves higher than the design wave height is progressive, but the displacement of several individual armor units will not necessarily result in the complete loss of protection. A logic diagram for the evaluation of the marine environment presented in Figure 7-6 summarizes the factors involved in selecting the design water depth and wave conditions to be used in the analysis of a rubble structure. The most severe wave condition for design of any part of a rubble-mound structure is usually the combination of predicted water depth and extreme incident wave height and period that produces waves which would break directly on the part of interest.

If a structure with two opposing slopes, such as a breakwater or jetty, will not be overtopped, a different design wave condition may be required for each side. The wave action directly striking one side of a structure, such as the harbor side of a breakwater, may be much less severe than that striking the other side. If the structure is porous enough to allow waves to pass through it, more armor units may be dislodged from the sheltered side's armor layer by waves traveling through the structure than by waves striking the layer directly. In such a case, the design wave for the sheltered side might be the same as for the exposed side, but no dependable analytical method is known for choosing such a design wave condition or for calculating a stable armor weight for it. Leeside armor sizes have been investigated in model tests by Markle (1982).

If a breakwater is designed to be overtopped, or if the designer is not sure that it will not be overtopped the crest and perhaps, the leeward side must be designed for breaking wave impact. Lording and Scott (1971) tested an overtopped rubble-mound structure that was subjected to breaking waves in water levels up to the crest elevation. Maximum damage to the leeside armor units occurred with the still-water level slightly below the crest and with waves breaking as close as two breaker heights from the toe of the structure. This would imply that waves were breaking over the structure and directly on the lee slope rather than on the seaward slope.
The crest of a structure designed to be submerged, or that might be submerged by hurricane storm surge, will undergo the heaviest wave action when the crest is exposed in the trough of a wave. The highest wave which would expose the crest can be estimated by using Figure 7-69, with the range of depths at the structure \( d \), the range of wave heights \( H \), and period \( T \), and the structure height \( h \). Values of \( \eta_c/H \) where \( \eta_c \) is the crest elevation above the still-water level, can be found by entering Figure 7-69 with \( H/gT^2 \) and \( d/gT^2 \). The largest breaking and nonbreaking wave heights for which

\[
d \leq h + H - \eta_c \tag{7-114}
\]

can then be used to estimate which wave height requires the heaviest armor. The final design breaking wave height can be determined by entering Figure 7-69 with values of \( d/gT^2 \) finding values of \( \eta_c/H \) for breaking conditions, and selecting the highest breaking wave which satisfied the equation

\[
d = h + H - \eta_c \tag{7-115}
\]

A structure that is exposed to a variety of water depths, especially a structure perpendicular to the shore, such as a groin, should have wave conditions investigated for each range of water depths to determine the highest breaking wave to which any part of the structure will be exposed. The outer end of a groin might be exposed only to wave forces on its sides under normal depths, but it might be overtopped and eventually submerged as a storm surge approaches. The shoreward end might normally be exposed to lower breakers, or perhaps only to broken waves. In the case of a high rubble-mound groin (i.e., a varying crest elevation and a sloping beach), the maximum breaking wave height may occur inshore of the seaward end of the groin.

c. **Hydraulics of Cover Layer Design.** Until about 1930, design of rubble structures was based only on experience and general knowledge of site conditions. Empirical formulas that subsequently developed are generally expressed in terms of the stone weight required to withstand design wave conditions. These formulas have been partially substantiated in model studies. They are guides and must be used with experience and engineering judgment. Physical modeling is often a cost-effective measure to determine the final cross-section design for most costly rubble-mound structures.

Following work by Iribarren (1938) and Iribarren and Nogales Y Olano (1950), comprehensive investigations were made by Hudson (1953, 1959, 1961a, and 1961b) at the U.S. Army Engineer Waterways Experiment Station (WES), and a formula was developed to determine the stability of armor units on rubble structures. The stability formula, based on the results of extensive small-scale model testing and some preliminary verification by large-scale model testing, is
where

\[ w = \text{weight in newtons or pounds of an individual armor unit in the primary cover layer.} \]

(When the cover layer is two quarrystones in thickness) the stones comprising the primary cover layer can range from about 0.75 \( W \) to 1.25 \( W \), with about 50 percent of the individual stones weighing more than \( W \). The gradation should be uniform across the face of the structure, with no pockets of smaller stone. The maximum weight of individual stones depends on the size or shape of the unit. The unit should not be of such a size as to extend an appreciable distance above the average level of the slope)

\[ w_r = \text{unit weight (saturated surface dry) of armor unit in N/m}^3 \text{ or lb/ft}^3 \text{. Note: Substitution of } \rho_r, \text{ the mass density of the armor material in kg/m}^3 \text{ or slugs/ft}^3, \text{ will yield } W \text{ in units of mass (kilograms or slugs)} \]

\[ H = \text{design wave height at the structure site in meters or feet (see Sec. III,7,b)} \]

\[ S_r = \text{specific gravity of armor unit, relative to the water at the structure } \left( S_r = w_r / w_w \right) \]

\[ w_w = \text{unit weight of water: fresh water } = 9,800 \text{ N/m}^3 \left( 62.4 \text{ lb/ft}^3 \right) \text{ seawater } = 10,047 \text{ N/m}^3 \left( 64.0 \text{ lb/ft}^3 \right) \text{ Note: Substitution of } \left( \frac{P_r - P_w}{P_w} \right)^3, \text{ where } P_w \text{ is the mass density of water at the structure for } (S_r - 1)^3, \text{ yields the same result} \]

\[ \theta = \text{angle of structure slope measured from horizontal in degrees} \]

\[ K_D = \text{stability coefficient that varies primarily with the shape of the armor units, roughness of the armor unit surface, sharpness of edges, and degree of interlocking obtained in placement (see Table 7-8).} \]

Equation 7-116 is intended for conditions when the crest of the structure is high enough to prevent major overtopping. Also the slope of the cover layer will be partly determined on the basis of stone sizes economically available. Cover layer slopes steeper than 1 on 1.5 are not recommended by the Corps of Engineers.

Equation 7-116 determines the weight of an armor unit of nearly uniform size. For a graded riprap armor stone, Hudson and Jackson (1962) have modified the equation to:
Table 7-8. Suggested $K_D$ Values for use in determining armor unit weight\(^1\).

<table>
<thead>
<tr>
<th>Armor Units</th>
<th>( n )</th>
<th>Placement</th>
<th>Structure Trunk</th>
<th>Structure Head</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>( K_D^2 ) Breaking Wave</td>
<td>( K_D ) Nonbreaking Wave</td>
<td>Breaking Wave</td>
</tr>
</tbody>
</table>

| Quarrystone Smooth rounded | 2     | Random    | 1.2 | 2.4 | 1.1 | 1.9 | 1.5 to 3.0 |
| Quarrystone Smooth rounded | >3    | Random    | 1.8 | 2.2 | 1.4 | 2.8 | 5 |
| Quarrystone Rough angular  | 1     | Random    | 2.0 | 4.0 | 1.6 | 2.8 | 2.0 |
| Quarrystone Rough angular  | >3    | Random    | 2.2 | 4.0 | 1.6 | 2.8 | 2.0 |
| Quarrystone Parallelepiped | 2     | Special 6 | 3.8 | 7.0 | 6.8 | 6.4 | 6.0 |
| Quarrystone Parallelepiped | 2     | Special 1 | 7.0 - 20.0 | 8.5 - 24.0 | -- | -- | -- |
| Tetrapod and Quadrupod     | 2     | Random    | 7.0 | 8.0 | 4.0 | 5.5 | 3.0 |
| Tribar                     | 2     | Random    | 9.0 | 10.0 | 6.0 | 6.5 | 6.0 |
| Dolos                      | 2     | Random    | 15.8\(^8\) | 31.8\(^8\) | 8.0 | 18.0 | 2.0\(^9\) |
| Modified cube              | 2     | Random    | 8.4 | 7.5 | --  | -- | 5.0 |
| Hexapod                    | 2     | Random    | 8.0 | 9.5 | 6.0 | 7.0 | 5.0 |
| Torskane                   | 2     | Random    | 11.0 | 22.0 | -- | -- | 5.0 |
| Tribar                     | 1     | Uniform   | 12.0 | 15.0 | 7.5 | 9.5 | 5.0 |
| Quarrystone \( (K_{R0}) \) |       | -- Random | 2.2 | 2.5 | -- | -- | -- |

\(^1\) **CAUTION:** Those \( K_D \) values shown in *italics* are unsupported by test results and are only provided for preliminary design purposes.

2. Applicable to slopes ranging from 1 on 1.5 to 1 on 5.

3. \( n \) is the number of units comprising the thickness of the armor layer.

4. The use of single layer of quarrystone armor units is not recommended for structures subject to breaking waves, and only under special conditions for structures subject to nonbreaking waves. When it is used, the stone should be carefully placed.

5. Until more information is available on the variation of \( K_D \) value with slope, the use of \( K_D \) should be limited to slopes ranging from 1 on 1.5 to 1 on 3. Some armor units tested on a structure head indicate a \( K_D \)-slope dependence.

6. Special placement with long axis of stone placed perpendicular to structure face.

7. Parallelepiped-shaped stone: long slab-like stone with the long dimension about 3 times the shortest dimension (Markle and Davidson, 1979).

8. Refers to no-damage criteria (<5 percent displacement, rocking, etc.); if no rocking (<2 percent) is desired, reduce \( K_D \) 50 percent (Zwamborn and Van Niekerk, 1982).

9. Stability of dolosse on slopes steeper than 1 on 2 should be substantiated by site-specific model tests.

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The symbols are the same as defined for equation (7-116). $W_{50}$ is the weight of the 50 percent size in the gradation. The maximum weight of graded rock is 4.0 ($W_{50}$); the minimum is 0.125 ($W_{50}$). Additional information on riprap gradation for exposure to wave forces is given by Ahrens (1981b). $K_{RR}$ is a stability coefficient for angular, graded riprap, similar to $K_D$. Values of $K_{RR}$ are shown in Table 7-8. These values allow for 5 percent damage (Hudson and Jackson, 1962).

Use of graded riprap cover layers is generally more applicable to revetments than to breakwaters or jetties. A limitation for the use of graded riprap is that the design wave height should be less than about 1.5 m (5 ft). For waves higher than 1.5 m (5 ft), it is usually more economical to use uniform-size armor units as specified by equation (7-116).

Values of $K_D$ and $K_{RR}$ are obtained from laboratory tests by first determining values of the stability number $N_s$ where

$$N_s = \frac{w_r^{1/3} H}{w_50^{1/3} (S_r-1)}$$

The stability number is plotted as a function of $cot \theta$ on log-log paper, and a straight line is fitted as a bottom envelope to the data such that

$$N_s = (K_D \ cot \theta)^{1/3} \ or \ (K_{RR} \ cot \theta)^{1/3}$$

Powers of $cot \theta$ other than 1/3 often give a better fit to the data. $N_s$ can be used for armor design by replacing $K_D \ cot \theta$ in equation (7-116) or $K_{RR} \ cot \theta$ in equation (7-117) with $N_s^3$, where $N_s$ is a function of some power of $cot \theta$.

d. Selection of Stability Coefficient. The dimensionless stability coefficient $K_D$ in equation (7-116) accounts for all variables other than structure slope, wave height, and the specific gravity of water at the site (i.e., fresh or salt water). These variables include:

1. Shape of armor units
2. Number of units comprising the thickness of armor layer
3. Manner of placing armor units
4. Surface roughness and sharpness of edges of armor units (degree of interlocking of armor units)
5. Type of wave attacking structure (breaking or nonbreaking)
(6) Part of structure (trunk or head)

(7) Angle of incidence of wave attack

(8) Model scale (Reynolds number)

(9) Distance below still-water level that the armor units extend down the face slope

(10) Size and porosity of underlayer material

(11) Core height relative to still-water level

(12) Crown type (concrete cap or armor units placed over the crown and extending down the back slope)

(13) Crown elevation above still-water level relative to wave height

(14) Crest width

Hudson (1959, 1961a, and 1961b), and Hudson and Jackson (1959), Jackson (1968a), Carver and Davidson (1977), Markle and Davidson (1979), Office, Chief of Engineers (1978), and Carver (1980) have conducted numerous laboratory tests with a view to establishing values of $K_D$ for various conditions of some of the variables. They have found that, for a given geometry of rubble structure, the most important variables listed above with respect to the magnitude of $K_D$ are those from (1) through (8). The data of Hudson and Jackson comprise the basis for selecting $K_D$, although a number of limitations in the application of laboratory results to prototype conditions must be recognized. These limitations are described in the following paragraphs.

(1) Laboratory waves were monochromatic and did not reproduce the variable conditions of nature. No simple method of comparing monochromatic and irregular waves is presently available. Laboratory studies by Deullet (1972) and Rogan (1969) have shown that action of irregular waves on model rubble structures can be modeled by monochromatic waves if the monochromatic wave height corresponds to the significant wave height of the spectrum of the irregular wave train. Other laboratory studies (i.e., Carstens, Traetteberg, and Tørum (1966); Brorsen, Burchard, and Larsen (1974); Feuillet and Sabaton (1980); and Tanimoto, Yagyu, and Goda (1982)) have shown, though, that the damage patterns on model rubble-mound structures with irregular wave action are comparable to model tests with monochromatic waves when the design wave height of the irregular wave train is higher than the significant wave height. As an extreme, the laboratory work of Feuillet and Sabaton (1980) and that of Tanimoto, Yagyu, and Goda (1982) suggest a design wave of $H_5$ when comparing monochromatic wave model tests to irregular wave model tests.

The validity of this comparison between monochromatic wave testing and irregular wave testing depends on the wave amplitude and phase spectra of the "groupiness" irregular wave train which, in turn, govern the groupiness of the wave train; i.e., the tendency of higher waves to occur together.

Groupiness in wave trains has been shown by Carstens, Traetteberg, and
Tørum (1966), Johnson, Mansard, and Ploeg (1978), and Burcharth (1979), to account for higher damage in rubble-mound or armor block structures. Burcharth (1979) found that grouped wave trains with maximum wave heights equivalent to monochromatic wave heights caused greater damage on dolosse- armored slopes than did monochromatic wave trains. Johnson, Mansard, and Ploeg (1978) found that grouped wave trains of energy density equivalent to that of monochromatic wave trains created greater damage on rubble-mound breakwaters.

Goda (1970b) and Andrew and Borgman (1981) have shown by simulation techniques that, for random-phased wave components in a wave spectrum, groupiness is dependent on the width of the spectral peak (the narrower the spectral width, the larger the groupiness in the wave train).

On a different tack, Johnson, Mansard, and Ploeg (1978) have shown that the same energy spectrum shape can produce considerably different damage patterns to a rubble-mound breakwater by controlling the phasing of the wave components in the energy spectrum. This approach to generating irregular waves for model testing is not presently attempted in most laboratories.

Typically, laboratory model tests assume random phasing of wave spectral components based on the assumption that waves in nature have random phasing. Tørum, Mathiesen, and Escutia (1979), Thompson (1981), Andrew and Borgman (1981), and Wilson and Baird (1972) have suggested that nonrandom phasing of waves appears to exist in nature, particularly in shallow water.

(2) Preliminary analysis of large-scale tests by Hudson (1975) has indicated that scale effects are relatively unimportant, and can be made negligible by the proper selection of linear scale to ensure that the Reynolds number is above $3 \times 10^4$ in the tests. The Reynolds number is defined in this case as

\[ R = \frac{(gH)^{1/2} k_{\Delta}}{v} \left( \frac{w}{w_p} \right)^{1/3} \]

where \( v \) is the kinematic viscosity of the water at the site and \( k_{\Delta} \) is the layer coefficient (see Sec. III,7,g(2)).

(3) The degree of interlocking obtained in the special placement of armor units in the laboratory is unlikely to be duplicated in the prototype. Above the water surface in prototype construction it is possible to place armor units with a high degree of interlocking. Below the water surface the same quality of interlocking can rarely be attained. It is therefore advisable to use data obtained from random placement in the laboratory as a basis for K values.

(4) Numerous tests have been performed for nonbreaking waves, but only limited test results are available for plunging waves. Values for these conditions were estimated based on breaking wave tests of similar armor units. The ratio between the breaking and nonbreaking wave K's for tetrapods and quadripods on structure trunks, for example, was used to estimate the breaking wave K's for tribars, modified cubes, and hexapods.
used on trunks. Similar comparisons of test results were used to estimate $K_D$ values for armor units on structure heads.

(5) Under similar wave conditions, the head of a rubble structure normally sustains more extensive and frequent damage than the trunk of the structure. Under all wave conditions, a segment of the slope of the rounded head of the structure is usually subject to direct wave attack regardless of wave direction. A wave trough on the lee side coincident with maximum runup on the windward side will create a high static head for flow through the structure.

(6) Sufficient information is not available to provide firm guidance on the effect of angle of wave approach on stability of armor units. Quarry-stone armor units are expected to show greater stability when subject to wave attack at angles other than normal incidence. However, an analysis of limited test results by Whillock (1977) indicates that dolos units on a 1-on-2 slope become less stable as the angle of wave attack increases from normal incidence ($0^\circ$) to approximately $45^\circ$. Stability increases rapidly again as the angle of wave attack increases beyond $45^\circ$. Whillock suggests that structures covered with dolosse should be designed only for the no-damage wave height at normal incidence if the structure is subject to angular wave attack. The stability of any rubble structures subjected to angular wave attack should be confirmed by hydraulic model tests.

Based on available data and the discussion above, Table 7-8 presents recommended values for $K_D$. Because of the limitations discussed, values in the table provide little or no safety factor. The values may allow some rocking of concrete armor units, presenting the risk of breakage. The $K_D$'s for dolosse may be reduced by 50 percent to protect against breakage, as noted in the footnote to Table 7-8. The experience of the field engineer may be utilized to adjust the $K_D$ value indicated in Table 7-8, but deviation to less conservative values is not recommended without supporting model test results. A two-unit armor layer is recommended. If a one-unit armor layer is considered, the $K_D$ values for a single layer should be obtained from Table 7-8. The indicated $K_D$ values are less for a single-stone layer than for a two-stone layer and will require heavier armor stone to ensure stability. More care must be taken in the placement of a single armor layer to ensure that armor units provide an adequate cover for the underlayer and that there is a high degree of interlock with adjacent armor units.

These coefficients were derived from large- and small-scale tests that used many various shapes and sizes of both natural and artificial armor units. Values are reasonably definitive and are recommended for use in design of rubble-mound structures, supplemented by physical model test results when possible.

The values given in Table 7-8 are indicated as no-damage criteria, but actually consider up to 5 percent damage. Higher values of percent damage to a rubble breakwater have been determined as a function of wave height for several of the armor unit shapes by Jackson (1968b). These values, together with statistical data concerning the frequency of occurrence of waves of different heights, can be used to determine the annual cost of maintenance as a function of the acceptable percent damage without endangering the functional characteristics of the structure. Knowledge of maintenance costs can be used
to choose a design wave height yielding the optimum combination of first and maintenance costs. A structure designed to resist waves of a moderate storm, but which may suffer damage without complete destruction during a severe storm may have a lower annual cost than one designed to be completely stable for larger waves.

Table 7-9 shows the results of damage tests where $H/H_D = 0$ is a function of the percent damage $D$ for various armor units. $H$ is the wave height corresponding to damage $D$. $H_D = 0$ is the design wave height corresponding to 0- to 5-percent damage, generally referred to as no-damage condition.

Table 7-9. $H/H_D = 0$ as a function of cover-layer damage and type of armor unit.\(^1\)

<table>
<thead>
<tr>
<th>Unit</th>
<th>Damage ($D$) in Percent</th>
<th>0 to 5</th>
<th>5 to 10</th>
<th>10 to 15</th>
<th>15 to 20</th>
<th>20 to 30</th>
<th>30 to 40</th>
<th>40 to 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarrystone (smooth)</td>
<td>$H/H_{D=0}$</td>
<td>1.00</td>
<td>1.08</td>
<td>1.14</td>
<td>1.20</td>
<td>1.29</td>
<td>1.41</td>
<td>1.54</td>
</tr>
<tr>
<td>Quarrystone (rough)</td>
<td>$H/H_{D=0}$</td>
<td>1.00</td>
<td>1.08</td>
<td>1.19</td>
<td>1.27</td>
<td>1.37</td>
<td>1.47</td>
<td>1.56(^2)</td>
</tr>
<tr>
<td>Tetrapods &amp; Quadrripods</td>
<td>$H/H_{D=0}$</td>
<td>1.00</td>
<td>1.09</td>
<td>1.17(^3)</td>
<td>1.24(^3)</td>
<td>1.32(^3)</td>
<td>1.41(^3)</td>
<td>1.50(^3)</td>
</tr>
<tr>
<td>Tribar</td>
<td>$H/H_{D=0}$</td>
<td>1.00</td>
<td>1.11</td>
<td>1.25(^3)</td>
<td>1.36(^3)</td>
<td>1.50(^3)</td>
<td>1.59(^3)</td>
<td>1.84(^3)</td>
</tr>
<tr>
<td>Dolos</td>
<td>$H/H_{D=0}$</td>
<td>1.00</td>
<td>1.10</td>
<td>1.14(^3)</td>
<td>1.17(^3)</td>
<td>1.20(^3)</td>
<td>1.24(^3)</td>
<td>1.27(^3)</td>
</tr>
</tbody>
</table>

\(^1\) Breakwater trunk, $n = 2$, random placed armor units, nonbreaking waves, and minor overtopping conditions.

\(^2\) Values in *italics* are interpolated or extrapolated.

\(^3\) **CAUTION**: Tests did not include possible effects of unit breakage. Waves exceeding the design wave height conditions by more than 10 percent may result in considerably more damage than the values tabulated.

The percent damage is based on the volume of armor units displaced from the breakwater zone of active armor unit removal for a specific wave height. This zone, as defined by Jackson (1968a), extends from the middle of the breakwater crest down the seaward face to a depth equivalent to one zero-damage wave height $H_D = 0$ below the still-water level. Once damage occurred, testing was continued for the specified wave condition until slope equilibrium was established or armor unit displacement ceased. Various recent laboratory tests on dolosse have indicated that once design wave conditions (i.e., zero-damage) are exceeded, damage progresses at a much greater rate than indicated.
from tests of other concrete armor units. Note from the table that waves producing greater than 10 percent damage to a dolos structure will produce lesser damage levels to structures covered with other armor units. Concrete units in general will fail more rapidly and catastrophically than quarrystone armor.

Caution must be exercised in using the values in Table 7-9 for breaking wave conditions, structure heads, or structures other than breakwaters or jetties. The damage zone is more concentrated around the still-water level on the face of a revetment than on a breakwater (Ahrens, 1975), producing deeper damage to the armor layer for a given volume of armor removed. As a result, damage levels greater than 30 percent signify complete failure of a revetment's armor. Model studies to determine behavior are recommended whenever possible.

The following example illustrates the ways in which Table 7-9 may be used.

***EXAMPLE PROBLEM 38***

**GIVEN:** A two-layer quarrystone breakwater designed for nonbreaking waves and minor overtopping from a no-damage design wave $H_D = 0 = 2.5$ m (8.2 ft) and $K_D = 4.0$.

**FIND:**

(a) The wave heights which would cause 5 to 10 percent, 10 to 15 percent, 15 to 20 percent, and 20 to 30 percent damage. The return periods of these different levels of damage and consequent repair costs could also be estimated, given appropriate long-term wave statistics for the site.

(b) The design wave height that should be used for calculating armor weight if the breakwater is a temporary or minor structure and 5 to 10 percent damage can be tolerated from 2.5-m waves striking it.

(c) The damage to be expected if stone weighing 75 percent of the zero-damage weight is available at substantially less cost or must be used in an emergency for an expedient structure.

**SOLUTION:**

(a) From Table 7-9, for rough quarrystone:

<table>
<thead>
<tr>
<th>Damage Level, %</th>
<th>0-5</th>
<th>5-10</th>
<th>10-15</th>
<th>15-20</th>
<th>20-30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H/H_D = 0$</td>
<td>1.00</td>
<td>1.08</td>
<td>1.19</td>
<td>1.27</td>
<td>1.37</td>
</tr>
<tr>
<td>$H$, m</td>
<td>2.5</td>
<td>2.7</td>
<td>3.0</td>
<td>3.2</td>
<td>3.4</td>
</tr>
</tbody>
</table>
Therefore, for instance, \( H_D = 5-10 = (2.5) (1.08) = 2.7 \text{ m (88.8 ft)} \).

(b) From Table 7-9, for \( D = 5 \) to 10 percent

\[
\frac{H}{H_{D=0}} = 1.08
\]

\[ H_{D=0} = \frac{H}{1.08} \]

Since the \( H \) causing 5 to 10 percent damage is 2.5 m

\[ H_{D=0} = \frac{2.5}{1.08} = 2.3 \text{ m (7.5 ft)} \]

(c) To determine the damage level, a ratio of wave heights must be calculated. The higher wave height "\( H \)" will be the \( H_{D=0} \) for the zero-damage weight \( W_{D=0} \). The lower wave height "\( H_{D=0} \)" will be the \( H_{D=0} \) for the available stone weight \( W_{AV} \).

Rearranging equation (7-116),

\[
H = (S_r -1) \left( \frac{W_{K_D \text{ cot } \theta}}{W_r} \right)^{1/3}
\]

from which

\[
\frac{"H"}{H_{D=0}} = \left( \frac{W_{D=0}}{W_{AV}} \right)^{1/3}
\]

Since \( W_{AV} = 0.75 \ W_{D=0} \)

\[
\frac{"H"}{H_{D=0}} = \left( \frac{W_{D=0}}{0.75 \ W_{D=0}} \right)^{1/3} = \left( \frac{1}{0.75} \right)^{1/3} = 1.10
\]

This corresponds to damage of about 5 to 10 percent if the available stone is used.

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

e. Importance of Unit Weight of Armor Units. The basic equation used for design of armor units for rubble structures indicates that the unit weight \( w_r \) of quarriystone or concrete is important. Designers should carefully evaluate the advantages of increasing unit weight of concrete armor units to affect savings in the structure cost. Brandtzaeg (1966) cautioned that variations in unit weight should be limited within a range of, say, 18.9
kilonewtons per cubic meter (120 pounds per cubic foot) to 28.3 kilonewtons per cubic meter (180 pounds per cubic foot). Unit weight of quarystone available from a particular quarry will likely vary over a narrow range of values. The unit weight of concrete containing normal aggregates is usually between 22.0 kilonewtons per cubic meter (140 pounds per cubic foot) and 24.3 kilonewtons per cubic meter (155 pounds per cubic foot). It can be made higher or lower through use of special heavy or lightweight aggregates that are usually available but are more costly than normal aggregates. The unit weight obtainable from a given set of materials and mixture proportions can be computed from Method CRD-3 of the Handbook for Concrete and Cement published by the U.S. Army Engineer Waterways Experiment Station (1949).

The effect of varying the unit weight of concrete is illustrated by the following example problem.

**EXAMPLE PROBLEM 39**

**GIVEN:** A 33.5-metric ton (36.8-short ton) concrete armor unit is required for the protection of a rubble-mound structure against a given wave height in salt water ($w_w = 10.0$ kilonewtons per cubic meter (64 pounds per cubic foot)). This weight was determined using a unit weight of concrete $w_r = 22.8$ kilonewtons per cubic meter (145 pounds per cubic foot).

**FIND:** Determine the required weight of armor unit for concrete with $w_r = 22.0$ kilonewtons per cubic meter (140 pounds per cubic foot) and $w_r = 26.7$ kilonewtons per cubic meter (170 pounds per cubic foot).

**SOLUTION:** Based on equation (7-116), the ratio between the unknown and known armor weight is

\[
\frac{w_r}{w_w} = \left(\frac{w_r}{w_w} - 1\right)^3
\]

Thus, for $w_r = 22.0$ kilonewtons per cubic meter

\[
\frac{22.0}{10.0} \left(\frac{22.0}{10.0} - 1\right)^3 = 33.5 \times \frac{12.7}{10.9} = 39.0 \text{ mt (42.9 tons)}
\]

For $w_r = 26.7$ kN/m$^3$

\[
\frac{26.7}{10.0} \left(\frac{26.7}{10.0} - 1\right)^3 = 33.5 \times \frac{5.7}{10.9} = 17.5 \text{ mt (19.2 tons)}
\]

**Concrete Armor Units.** Many different concrete shapes have been developed as armor units for rubble structures. The major advantage of
concrete armor units is that they usually have a higher stability coefficient value and thus permit the use of steeper structure side slopes or a lighter weight of armor unit. This advantage has particular value when quarystone of the required size is not available.

Table 7-10 lists the concrete armor units that have been cited in literature and shows where and when the unit was developed. One of the earlier nonblock concrete armor units was the *tetrapod*, developed and patented in 1950 by Neyrpc, Inc., of France. The tetrapod is an unreinforced concrete shape with four truncated conical legs projecting radially from a center point (see Fig. 7-108).

Figure 7-109 provides volume, weight, dimensions, number of units per 1000 square feet, and thickness of layers of the tetrapod unit. The *quadrripod* (Fig. 7-108) was developed and tested by the United States in 1959; details are shown in Figure 7-110.

In 1958, R. Q. Palmer, United States, developed and patented the *tribar*. This concrete shape consists of three cylinders connected by three radial arms (see Fig. 7-108). Figure 7-111 provides details on the volume, dimensions, and thickness of layers of tribars.

The *dolos* armor unit, developed in 1963 by E. M. Merrifield, Republic of South Africa (Merrifield and Zwamborn, 1966), is illustrated in Figure 7-108. This concrete unit closely resembles a ship anchor or an "H" with one vertical perpendicular to the other. Detailed dimensions are shown in Figure 7-112.

The *toskane* is similar to the dolos, but the shapes at the ends of the central shank are triangular heads rather than straight flukes. The triangular heads are purported to be more resistant to breakage than the dolos flukes. A round hole may be placed through each head to increase porosity. Dimensions are shown in Figure 7-113.

As noted in Table 7-8, various other shapes have been tested by the Corps of Engineers. Details of the *modified cube* and *hexapod* are shown in Figures 7-114 and 7-115, respectively.

As noted, the tetrapod, quadrripod, and tribar are patented, but the U.S. patents on these units have expired. Patents on these units may still be in force in other countries, however; payment of royalties to the holder of the patent for the use of such a unit is required. Since other units in Table 7-10 may be patented, in the U.S. or elsewhere, the status of patents should be reviewed before they are used.

Unlike quarystone, concrete armor units have a history of breakage problems. If a unit breaks, its weight is reduced; if enough units break, the stability of an armor layer is reduced. For dolosse, for instance, model tests by Markel and Davidson (1984a) have demonstrated that random breakage of up to 15 percent or up to 5 broken units in a cluster will have little effect on stability. Breakage exceeding these limits may lead to catastrophic failure of the armor layer.
Table 7-10. Types of armor units.¹

<table>
<thead>
<tr>
<th>Name of Unit</th>
<th>Development of Unit</th>
<th>Country/Date</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Akkon</td>
<td>Netherlands</td>
<td>1962</td>
<td>Pascoe and Walther, 1963</td>
</tr>
<tr>
<td>Binne block</td>
<td>England</td>
<td>1962</td>
<td>Hydraulics Research Station, 1980</td>
</tr>
<tr>
<td>Tripod</td>
<td>Netherlands</td>
<td>1962</td>
<td>Pascoe and Walther, 1963</td>
</tr>
<tr>
<td>Cube</td>
<td>4</td>
<td>1969</td>
<td>Hudson and Jackson, 1953</td>
</tr>
<tr>
<td>Cube (modified)</td>
<td>United States</td>
<td>1959</td>
<td>Jackson, 1968a</td>
</tr>
<tr>
<td>Dolos</td>
<td>South Africa</td>
<td>1963</td>
<td>Merrifield and Zamborn, 1966</td>
</tr>
<tr>
<td>Dom</td>
<td>Mexico</td>
<td>1970</td>
<td>--</td>
</tr>
<tr>
<td>Gosho block</td>
<td>Japan</td>
<td>1967</td>
<td>Personal correspondence, 1971, Prof. S. Negai, Dean of Faculty of Engineering, Osaka City University, Sugimoto-Chou, Shizuoka-Ka, Osaka, Japan</td>
</tr>
<tr>
<td>Grabholar</td>
<td>South Africa</td>
<td>1957</td>
<td>Personal correspondence, 1971, Mr. F. Grobellaar, Technical Manager, Fisheries Development Corp. of South Africa, Ltd., Cape Town, Republic of South Africa</td>
</tr>
<tr>
<td>Hexaleg block</td>
<td>Japan</td>
<td>--</td>
<td>Oikita Kogyo Co., Ltd., undated</td>
</tr>
<tr>
<td>Hexapod³</td>
<td>United States</td>
<td>1959</td>
<td>Jackson, 1968a</td>
</tr>
<tr>
<td>Hollow square</td>
<td>Japan</td>
<td>1960</td>
<td>Personal correspondence, 1971, Prof. S. Negai (see above); Negai, 1962.</td>
</tr>
<tr>
<td>Hollow tetrahedron</td>
<td>Japan</td>
<td>1959</td>
<td>Personal correspondence, 1971, Prof. S. Negai (see above); Negai, 1961b; Tamaka et al., 1960</td>
</tr>
<tr>
<td>Interlocking H-block</td>
<td>United States</td>
<td>1958</td>
<td>U. S. Army Engineer District, Galveston, 1972</td>
</tr>
<tr>
<td>Mexapod³</td>
<td>Mexico</td>
<td>1978</td>
<td>Portas and Medina, 1978</td>
</tr>
<tr>
<td>M-shaped block</td>
<td>United States</td>
<td>1960</td>
<td>Personal correspondence, 1971, Prof. S. Negai (see above); Negai, 1962.</td>
</tr>
<tr>
<td>Pelican stone²</td>
<td>United States</td>
<td>1960</td>
<td>Jackson, 1961</td>
</tr>
<tr>
<td>Quadrupod</td>
<td>United States</td>
<td>1959</td>
<td>Jackson, 1968a</td>
</tr>
<tr>
<td>Rectangular block³</td>
<td></td>
<td>4</td>
<td>Jackson, 1967</td>
</tr>
<tr>
<td>Rentrapod</td>
<td>England</td>
<td>--</td>
<td>Hydraulics Research Station, 1980</td>
</tr>
<tr>
<td>Seebee</td>
<td>Australia</td>
<td>1978</td>
<td>Brown, 1978</td>
</tr>
<tr>
<td>Stabilized</td>
<td>Romania</td>
<td>1965</td>
<td>Lates and Hiiusena, 1966</td>
</tr>
<tr>
<td>Sta-Per³</td>
<td>United States</td>
<td>1966</td>
<td>Personal correspondence, 1971, Mr. R. J. O'Kelliell, Marine Modules, Inc., Yonkers, N.Y.</td>
</tr>
<tr>
<td>Sta-Post³</td>
<td>United States</td>
<td>1966</td>
<td>Personal correspondence, 1971, Mr. R. J. O'Kelliell (see above)</td>
</tr>
<tr>
<td>Stakc cube</td>
<td>Netherlands</td>
<td>1965</td>
<td>Hakkeling, 1971</td>
</tr>
<tr>
<td>Stone block</td>
<td>Norway</td>
<td>1961</td>
<td>Sven, Trastenberg, and Térum, 1965</td>
</tr>
<tr>
<td>Tetrahedron (solid)³</td>
<td>United States</td>
<td>1959</td>
<td>Jackson, 1968a</td>
</tr>
<tr>
<td>Tetrahedron (perforated)³</td>
<td>United States</td>
<td>1959</td>
<td>Jackson, 1968a</td>
</tr>
<tr>
<td>Tetrapod³</td>
<td>France</td>
<td>1960</td>
<td>Denei, Chapue, and Chalille, 1960; Jackson, 1968</td>
</tr>
<tr>
<td>Yoskane³</td>
<td>South Africa</td>
<td>1966</td>
<td>Personal correspondence, 1971, Mr. F. Grobellaar (see above)</td>
</tr>
<tr>
<td>Tristar</td>
<td>United States</td>
<td>1958</td>
<td>Jackson, 1968a; Personal correspondence, Mr. Robert Q. Palmer, President, Tribeca, Inc., Las Vegas, Nevada</td>
</tr>
<tr>
<td>Trigon</td>
<td>United States</td>
<td>1962</td>
<td>--</td>
</tr>
<tr>
<td>Tri-Long</td>
<td>United States</td>
<td>1968</td>
<td>Davidson, 1971</td>
</tr>
<tr>
<td>Tripod</td>
<td>Netherlands</td>
<td>1962</td>
<td>Pascoe and Walther, 1963</td>
</tr>
<tr>
<td>Tripod block</td>
<td>England</td>
<td>1974</td>
<td>British Transport Docks Board, 1979</td>
</tr>
</tbody>
</table>

¹ Modified from Hudson, 1974.
² Not available.
³ Units have been tested, some extensively, at the U. S. Army Engineer Waterways Experiment Station (WES); not all units were tested in two-layer armor layers.
⁴ Cubes and rectangular blocks are known to have been used in masonry-type breakwaters since early Roman times and in rubble-mound breakwaters during the last few centuries. The cube was tested at WES as a construction block for breakwaters as early as 1943.
⁵ Solid tetrahedrons are known to have been used in hydraulic works for many years. This unit was tested at WES in 1959.
Figure 7-108. Views of the tetrapod, quadripod, tribar, and dolos armor units.
Figure 7-109. Tetrapod specifications.
Figure 7-110. Quadripod specifications.
Figure 7-111. Tribar specifications.
Figure 7-112. Dolos specifications.
VOLUME OF INDIVIDUAL ARMOR UNIT \( (V) = 0.083H^3 \)

where:
\[
\begin{align*}
A &= 0.575H \\
B &= 0.260H \\
C &= 0.345H \\
D &= 0.310H \\
H &= \text{overall height of unit}
\end{align*}
\]

ARMOR LAYER THICKNESS (2 UNITS) = 0.899\(H\)

NUMBER OF TOSKANES (TWO LAYERS, RANDOM PLACED)

PER UNIT AREA \( (N_i) = 0.99V^{2/3} \) (see eq. (7.122))

where:
\[
\begin{align*}
k_\Delta &= 1.03 \\
P &= 52
\end{align*}
\]

Figure 7-113. Toskane specifications.
Figure 7-114. Modified cube specifications.
Figure 7-115. Hexapod specifications.

**Volume of Individual Armor Units (cu ft)**

<table>
<thead>
<tr>
<th>Unit</th>
<th>7.14</th>
<th>16.29</th>
<th>28.57</th>
<th>71.63</th>
<th>162.86</th>
<th>214.29</th>
<th>285.71</th>
<th>357.16</th>
<th>428.57</th>
<th>500.00</th>
<th>571.43</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (lb/cu ft)</td>
<td>0.90</td>
<td>1.00</td>
<td>2.00</td>
<td>5.00</td>
<td>10.00</td>
<td>15.00</td>
<td>20.00</td>
<td>25.00</td>
<td>30.00</td>
<td>35.90</td>
<td>40.00</td>
</tr>
</tbody>
</table>

**Average Measured Thickness of One Layer Placed Uniformly (ft)**

- 2.40
- 3.13
- 3.94
- 5.18
- 6.74
- 7.72
- 8.90
- 9.15
- 9.73
- 10.24
- 10.70

**Average Measured Thickness of Two Layers Random Placed (ft)**

- 4.03
- 5.56
- 5.13
- 5.46
- 6.02
- 7.76
- 9.15
- 9.20
- 10.16
- 10.26
- 9.06

**Number of Armor Units Per 1000 sq ft (One Layer Placed Uniformly)**

- 199.03
- 124.70
- 78.60
- 42.72
- 26.84
- 20.51
- 16.99
- 14.83
- 13.01
- 11.69
- 10.74

**Number of Armor Units Per 1000 sq ft (Two Layers Random Placed)**

- 328.40
- 206.66
- 130.31
- 70.82
- 44.99
- 34.01
- 26.16
- 24.26
- 21.58
- 19.38
- 17.00

**Dimensions of Armor Units (ft)**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.23</td>
<td>1.55</td>
<td>1.95</td>
<td>2.65</td>
<td>3.34</td>
<td>3.82</td>
<td>4.21</td>
<td>4.53</td>
<td>4.82</td>
<td>5.07</td>
<td>5.30</td>
</tr>
<tr>
<td>B</td>
<td>1.11</td>
<td>1.40</td>
<td>1.76</td>
<td>2.39</td>
<td>3.01</td>
<td>3.45</td>
<td>3.80</td>
<td>4.09</td>
<td>4.35</td>
<td>4.57</td>
<td>4.78</td>
</tr>
<tr>
<td>C</td>
<td>0.74</td>
<td>0.93</td>
<td>1.17</td>
<td>1.59</td>
<td>2.01</td>
<td>2.30</td>
<td>2.73</td>
<td>2.90</td>
<td>3.09</td>
<td>3.19</td>
<td></td>
</tr>
</tbody>
</table>

**Volume of Individual Armor Unit (cu ft)**

- 0.176

**Note:** Data based on hexapods used in model tests conducted at the Waterways Experiment Station.
Two approaches have been proposed to control breakage. Zwamborn and Van Niekerk, (1981, 1982) surveyed the performance of dolos-armored breakwaters worldwide and concluded that most structures that failed had been under-designed or had experienced construction difficulties. They formulated lower values for the stability coefficients to produce heavier armor units which would be stable against any crack-causing movement such as rocking in place under wave action. Their results are reflected in Table 7-8. Reinforcement of units with steel bar and fibers (Magoon and Shimizer, 1971) has been tried on several structures. Markle and Davidson (1984b) have surveyed the breakage of reinforced and unreinforced armor units on Corps structures and have found field tests to be inconclusive. No proven analytical method is known for predicting what wave conditions will cause breakage or what type or amount of reinforcement will prevent it.

Projects using tetrapods, tribars, quadripods, and dolosse in the United States are listed in Table 7-11.

g. Design of Structure Cross-Section. A rubble structure is normally composed of a bedding layer and a core of quarry-run stone covered by one or more layers or larger stone and an exterior layer(s) of large quarrystone or concrete armor units. Typical rubble-mound cross sections are shown in Figures 7-116 and 7-117. Figure 7-116 illustrates cross-section features typical of designs for breakwaters exposed to waves on one side (seaward) and intended to allow minimal wave transmission to the other (leeward) side. Breakwaters of this type are usually designed with crests elevated such that overtopping occurs only in very severe storms with long return periods. Figure 7-117 shows features common to designs where the breakwater may be exposed to substantial wave action from both sides, such as the outer portions of jetties, and where overtopping is allowed to be more frequent. Both figures show both a more complex "idealized" cross section and a "recommended" cross section. The idealized cross section provides more complete use of the range of materials typically available from a quarry, but is more difficult to construct. The recommended cross section takes into account some of the practical problems involved in constructing submerged features.

The right-hand column of the table in these figures gives the rock-size gradation of each layer as a percent of the average layer rock size given in the left-hand column. To prevent smaller rocks in an underlayer from being pulled through an overlayer by wave action, the following criterion for filter design (Sowers and Sowers, 1970) may be used to check the rock-size gradations given in Figures 7-116 and 7-117.

\[ D_{15} \text{ (cover)} \leq 5 \times D_{85} \text{ (under)} \]

where \( D_{85} \text{ (under)} \) is the diameter exceeded by the coarsest 15 percent of the underlayer and \( D_{15} \text{ (cover)} \) is the diameter exceeded by the coarsest 85 percent of the layer immediately above the underlayer.
<table>
<thead>
<tr>
<th>Date</th>
<th>Location</th>
<th>Structure Type</th>
<th>Construction Type</th>
<th>Armor Type and Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1930 - 1958</td>
<td>Humboldt Bay, Calif.¹</td>
<td>North and south jetties</td>
<td>Rehabilitation</td>
<td>100-ton concrete blocks and 12-ton tetrahedrons, unreb.</td>
</tr>
<tr>
<td>1957</td>
<td>Crescent City, Calif.¹</td>
<td>Breakwater</td>
<td>Original</td>
<td>25-ton tetrapods, unreb.</td>
</tr>
<tr>
<td>1957</td>
<td>Rincon Island, Calif.⁵</td>
<td>Seawall</td>
<td>Original</td>
<td>31-ton tetrapods, unreb.</td>
</tr>
<tr>
<td>1959</td>
<td>Nāwiliwili, Kauai, Hawaii¹</td>
<td>Breakwater</td>
<td>Rehabilitation</td>
<td>17.8-ton trirms, reinforced</td>
</tr>
<tr>
<td>1960 - 1963</td>
<td>Humboldt Bay, Calif.¹</td>
<td>North and south jetties</td>
<td>Rehabilitation</td>
<td>20- to 100-ton concrete blocks, unreb.</td>
</tr>
<tr>
<td>1963</td>
<td>Santa Cruz, Calif.¹</td>
<td>West jetty</td>
<td>Original</td>
<td>28-ton quadrupods, unreb.</td>
</tr>
<tr>
<td>1966</td>
<td>Ventura Harbor, Calif.¹</td>
<td>Jetty</td>
<td>Original</td>
<td>10.7-ton trims, unreb.</td>
</tr>
<tr>
<td>1969</td>
<td>Kahului Harbor, Maui, Hawaii¹</td>
<td>Breakwater</td>
<td>Rehabilitation</td>
<td>35- to 50-ton trims, reinforced</td>
</tr>
<tr>
<td>1971 - 1972</td>
<td>Humboldt Bay, Calif.¹</td>
<td>North and south jetties</td>
<td>Rehabilitation</td>
<td>19-ton trims, reinforced</td>
</tr>
<tr>
<td>1972</td>
<td>Diablo Canyon, Calif.²</td>
<td>Breakwater</td>
<td>Original</td>
<td>42- to 43-ton dolosse, reinforced</td>
</tr>
<tr>
<td>1973</td>
<td>Kahului Harbor, Maui, Hawaii¹</td>
<td>Breakwater</td>
<td>Rehabilitation</td>
<td>24.5- to 37.1-ton trims, unreb.</td>
</tr>
<tr>
<td>1974</td>
<td>Crescent City, Calif.¹</td>
<td>Breakwater</td>
<td>Rehabilitation</td>
<td>19- to 35-ton trims, reinforced</td>
</tr>
<tr>
<td>1975</td>
<td>Honolulu Airport, Oahu, Hawaii³</td>
<td>Seawall</td>
<td>Original</td>
<td>40-ton dolosse, unreb.</td>
</tr>
<tr>
<td>1977</td>
<td>Kahului Harbor, Maui, Hawaii¹</td>
<td>East and west breakwater</td>
<td>Rehabilitation</td>
<td>4- to 6-ton dolosse, unreb.</td>
</tr>
<tr>
<td>1977</td>
<td>Nāwiliwili, Kauai, Hawaii¹</td>
<td>Breakwater</td>
<td>Rehabilitation</td>
<td>20- to 30-ton dolosse, reinforced, and 6-ton dolosse, unreb.</td>
</tr>
<tr>
<td>1979</td>
<td>Pohoihi Bay, Hawaii, Hawaii¹</td>
<td>Breakwater</td>
<td>Original</td>
<td>11-ton dolosse, unreb.</td>
</tr>
<tr>
<td>1979</td>
<td>Waianae Harbor, Oahu, Hawaii¹</td>
<td>Breakwater</td>
<td>Original</td>
<td>6-ton dolosse, unreb.</td>
</tr>
<tr>
<td>1980</td>
<td>Manasquan Inlet, N.J.¹</td>
<td>Jetty</td>
<td>Rehabilitation</td>
<td>2-ton dolosse, unreb.</td>
</tr>
<tr>
<td>1980</td>
<td>Cleveland Harbor, Ohio¹</td>
<td>Breakwater</td>
<td>Rehabilitation</td>
<td>16-ton dolosse, reinforced</td>
</tr>
<tr>
<td>1982</td>
<td>Cleveland Harbor, Ohio¹</td>
<td>Breakwater</td>
<td>Rehabilitation</td>
<td>2-ton dolosse, unreb.</td>
</tr>
<tr>
<td>1983</td>
<td>International Airport, St. Thomas, Virgin Islands⁴</td>
<td>Revetment</td>
<td>Original</td>
<td>6- to 10-ton dolosse, unreb.</td>
</tr>
</tbody>
</table>

¹ Markle and Davidson (1984b).
² Lillevang (1977).
³ Darling, (1976).
⁴ Czerniak, Lord, and Collins (1979) and personal communication with Earle Howard, U. S. Army Engineer District, Jacksonville, Fla., 1983.
⁵ Keith and Skjel (1974).
Figure 7-116. Rubble-mound section for seaward wave exposure with zero-to-moderate overtopping conditions.
Figure 7-117. Rubble-mound section for wave exposure on both sides with moderate overtopping conditions.
Stone sizes are given by weight in Figures 7-116 and 7-117 since the armor in the cover layers is selected by weight at the quarry, but the smaller stone sizes are selected by dimension using a sieve or a grizzly. Thomsen, Wohlt, and Harrison (1972) found that the sieve size of stone $w^{1/3} W$ corresponds approximately to $0.15\frac{w_r}{W}$, where $W$ is the stone weight and $w_r$ is the stone unit weight, both in the same units of mass or force. As an aid to understanding the stone sizes referenced in Figures 7-116 and 7-117, Table 7-12 lists weights and approximate dimensions of stones of 25.9 kilonewtons per cubic meter (165 pounds per cubic foot) unit weight. The dimension given for stone weighing several tons is approximately the size the stone appears to visual inspection. Multiples of these dimensions should not be used to determine structure geometry since the stone intermeshes when placed.

A logic diagram for the preliminary design of a rubble structure is shown in Figure 7-118. The design can be considered in three phases: (1) structure geometry, (2) evaluation of construction technique, and (3) evaluation of design materials. A logic diagram for evaluation of the preliminary design is shown in Figure 7-119.

As part of the design analysis indicated in the logic diagram (Fig. 7-118), the following structure geometry should be investigated:

1. Crest elevation and width.
2. Concrete cap for rubble-mound structures.
3. Thickness of armor layer and underlayers and number of armor units.
4. Bottom elevation of primary cover layer.
5. Toe berm for cover layer stability.
6. Structure head and lee side cover layer.
7. Secondary cover layer.
8. Underlayers.
10. Scour protection at toe.
11. Toe berm for foundation stability.

(1) **Crest Elevation and Width.** Overtopping of a rubble structure such as a breakwater or jetty usually can be tolerated only if it does not cause damaging waves behind the structure. Whether overtopping will occur depends on the height of the wave runup $R$. Wave runup depends on wave characteristics, structure slope, porosity, and roughness of the cover layer. If the armor layer is chinked, or in other ways made smoother or less permeable—as a graded riprap slope—the limit of maximum riprap will be
Table 7-12. Weight and size selection dimensions of quarrrystone.

<table>
<thead>
<tr>
<th>Weight (ton)</th>
<th>Dimension (m)</th>
<th>Weight (kg)</th>
<th>Dimension (m)</th>
<th>Weight (lb)</th>
<th>Dimension (cm)</th>
<th>Weight (kg)</th>
<th>Dimension (cm)</th>
<th>Weight (lb)</th>
<th>Dimension (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.907 (1)</td>
<td>0.81 (2.64)</td>
<td>45.36 (100)</td>
<td>0.30 (0.97)</td>
<td>2.27 (5)</td>
<td>10.92 (4.30)</td>
<td>0.23 (0.5)</td>
<td>5.08 (2.00)</td>
<td>0.01 (0.025)</td>
<td>1.88 (0.74)</td>
</tr>
<tr>
<td>1.814 (2)</td>
<td>1.02 (3.33)</td>
<td>90.72 (200)</td>
<td>9.38 (1.23)</td>
<td>4.54 (10)</td>
<td>13.77 (5.42)</td>
<td>0.45 (1.0)</td>
<td>6.40 (2.52)</td>
<td>0.02 (0.050)</td>
<td>2.36 (0.93)</td>
</tr>
<tr>
<td>2.722 (3)</td>
<td>1.16 (3.81)</td>
<td>136.08 (300)</td>
<td>0.43 (1.40)</td>
<td>6.81 (15)</td>
<td>15.77 (6.21)</td>
<td>0.68 (1.5)</td>
<td>7.32 (2.88)</td>
<td>0.03 (0.075)</td>
<td>2.70 (1.06)</td>
</tr>
<tr>
<td>3.629 (4)</td>
<td>1.28 (4.19)</td>
<td>181.44 (400)</td>
<td>9.50 (1.54)</td>
<td>9.07 (20)</td>
<td>17.35 (6.83)</td>
<td>0.91 (2.0)</td>
<td>8.05 (3.17)</td>
<td>0.04 (0.100)</td>
<td>2.97 (1.17)</td>
</tr>
<tr>
<td>4.536 (5)</td>
<td>1.38 (4.52)</td>
<td>226.80 (500)</td>
<td>0.51 (1.66)</td>
<td>11.34 (25)</td>
<td>18.70 (7.36)</td>
<td>1.13 (2.5)</td>
<td>8.66 (3.41)</td>
<td>0.06 (0.125)</td>
<td>3.20 (1.26)</td>
</tr>
<tr>
<td>5.443 (6)</td>
<td>1.46 (4.80)</td>
<td>272.16 (600)</td>
<td>0.54 (1.77)</td>
<td>13.61 (30)</td>
<td>19.86 (7.82)</td>
<td>1.36 (3.0)</td>
<td>9.22 (3.63)</td>
<td>0.07 (0.130)</td>
<td>3.40 (1.34)</td>
</tr>
<tr>
<td>6.350 (7)</td>
<td>1.54 (5.05)</td>
<td>317.52 (700)</td>
<td>0.57 (1.86)</td>
<td>15.88 (35)</td>
<td>20.90 (8.23)</td>
<td>1.59 (3.5)</td>
<td>9.70 (3.82)</td>
<td>0.08 (0.175)</td>
<td>3.58 (1.41)</td>
</tr>
<tr>
<td>7.258 (8)</td>
<td>1.61 (5.28)</td>
<td>362.88 (800)</td>
<td>0.60 (1.95)</td>
<td>18.14 (40)</td>
<td>21.84 (8.60)</td>
<td>1.81 (4.0)</td>
<td>10.13 (3.99)</td>
<td>0.09 (0.200)</td>
<td>3.73 (1.47)</td>
</tr>
<tr>
<td>8.165 (9)</td>
<td>1.67 (5.49)</td>
<td>408.24 (900)</td>
<td>0.62 (2.02)</td>
<td>20.41 (45)</td>
<td>22.73 (8.95)</td>
<td>2.04 (4.5)</td>
<td>10.54 (4.15)</td>
<td>0.10 (0.225)</td>
<td>3.89 (1.53)</td>
</tr>
<tr>
<td>9.072 (10)</td>
<td>1.73 (5.69)</td>
<td>453.60 (1000)</td>
<td>0.64 (2.10)</td>
<td>22.68 (50)</td>
<td>23.55 (9.27)</td>
<td>2.27 (5.0)</td>
<td>10.92 (4.30)</td>
<td>0.11 (0.250)</td>
<td>4.04 (1.59)</td>
</tr>
</tbody>
</table>

1 Dimensions correspond to size measured by sieve, grizzly, or visual inspection for stone of 25.9 kilonewtons per cubic meter unit weight. Do not use for determining structure crest width or layer thickness.
Figure 7-118. Logic diagram for preliminary design of rubble structure.
Figure 7-119. Logic diagram for evaluation of preliminary design.
higher than for rubble slopes (see Section II,1, and Figs. 7-19 and 7-20). The selected crest elevation should be the lowest that provides the protection required. Excessive overtopping of a breakwater or jetty can cause choppiness of the water surface behind the structure and can be detrimental to harbor operations, since operations such as mooring of small craft and most types of commercial cargo transfer require calm waters. Overtopping of a rubble seawall or revetment can cause serious erosion behind the structure and flooding of the backshore area. Overtopping of jetties can be tolerated if it does not adversely affect the channel.

The width of the crest depends greatly on the degree of allowable overtopping; where there will be no overtopping, crest width is not critical. Little study has been made of crest width of a rubble structure subject to overtopping. Consider as a general guide for overtopping conditions that the minimum crest width should equal the combined widths of three armor units \( (n = 3) \). Crest width may be obtained from the following equation.

\[
B = nk_\Delta \left( \frac{W}{w_r} \right)^{1/3}
\]

where

- \( B \) = crest width, m (or ft)
- \( n \) = number of stones \( (n = 3 \text{ is recommended minimum}) \)
- \( k_\Delta \) = layer coefficient (Table 7-13)
- \( W \) = mass of armor unit in primary cover layer, kg (or weight in lb)
- \( w_r \) = mass density of armor unit, kg/m\(^3\) (or unit weight in lb/ft\(^3\))

The crest should be wide enough to accommodate any construction and maintenance equipment which may be operated from the structure.

Figures 7-116 and 7-117 show the armor units of the primary cover layer, sized using equation (7-116), extended over the crest. Armor units of this size are probably stable on the crest for the conditions of minor to no overtopping occurring in the model tests which established the values of \( K_D \) in Table 7-8. Such an armor unit size can be used for preliminary design of the cross section of an overtopped or submerged structure, but model tests are strongly recommended to establish the required stable armor weight for the crest of a structure exposed to more than minor overtopping. Concrete armor units placed on the crest of an overtopped structure may be much less stable than the equivalent quarystone armor chosen using equation (7-116) on a structure with no overtopping. In the absence of an analytical method for calculating armor weight for severely overtopped or submerged structures, especially those armored with concrete units, hydraulic model tests are necessary. Markle and Carver (1977) have tested heavily overtopped and submerged quarystone-armored structures.
Table 7-13. Layer coefficient and porosity for various armor units.

<table>
<thead>
<tr>
<th>Armor Unit</th>
<th>n</th>
<th>Placement</th>
<th>Layer Coefficient $k_A$</th>
<th>Porosity (P) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarrystone (smooth)$^1$</td>
<td>2</td>
<td>Random</td>
<td>1.02</td>
<td>38</td>
</tr>
<tr>
<td>Quarrystone (rough)$^2$</td>
<td>2</td>
<td>Random</td>
<td>1.00</td>
<td>37</td>
</tr>
<tr>
<td>Quarrystone (rough)$^2$</td>
<td>&gt;3</td>
<td>Random</td>
<td>1.00</td>
<td>40</td>
</tr>
<tr>
<td>Quarrystone (parallelepiped)$^6$</td>
<td>2</td>
<td>Special</td>
<td>--</td>
<td>27</td>
</tr>
<tr>
<td>Cube (modified)$^1$</td>
<td>2</td>
<td>Random</td>
<td>1.10</td>
<td>47</td>
</tr>
<tr>
<td>Tetrapod$^1$</td>
<td>2</td>
<td>Random</td>
<td>1.04</td>
<td>50</td>
</tr>
<tr>
<td>Quadripod$^1$</td>
<td>2</td>
<td>Random</td>
<td>0.95</td>
<td>49</td>
</tr>
<tr>
<td>Hexipod$^1$</td>
<td>2</td>
<td>Random</td>
<td>1.15</td>
<td>47</td>
</tr>
<tr>
<td>Tribar$^1$</td>
<td>2</td>
<td>Random</td>
<td>1.02</td>
<td>54</td>
</tr>
<tr>
<td>Dolos$^4$</td>
<td>2</td>
<td>Random</td>
<td>0.94</td>
<td>56</td>
</tr>
<tr>
<td>Toskane$^5$</td>
<td>2</td>
<td>Random</td>
<td>1.03</td>
<td>52</td>
</tr>
<tr>
<td>Tribar$^1$</td>
<td>1</td>
<td>Uniform</td>
<td>1.13</td>
<td>47</td>
</tr>
<tr>
<td>Quarrystone$^7$</td>
<td></td>
<td>Graded</td>
<td>Random</td>
<td>--</td>
</tr>
</tbody>
</table>

1 Hudson (1974).
3 Hudson, (1961a).
4 Carver and Davidson (1977).
5 Carver (1978).
6 Layer thickness is twice the average long dimension of the parallelepiped stones. Porosity is estimated from tests on one layer of uniformly placed modified cubes (Hudson, 1974).
7 The minimum layer thickness should be twice the cubic dimension of the $W_{50}$ riprap. Check to determine that the graded layer thickness is $\geq 1.25$ the cubic dimension of the $W_{max}$ riprap (see eqs. 7-123 and 7-124 below).
(2) **Concrete Cap for Rubble-Mound Structures.** Placed concrete has been added to the cover layer of rubble-mound jetties and breakwaters. Such use ranges from filling the interstices of stones in the cover layer, on the crest, and as far down the slopes as wave action permits, to casting large monolithic blocks of several hundred kilograms. This concrete may serve any of four purposes: (a) to strengthen the crest, (b) to deflect overtopping waves away from impacting directly on the lee side slope, (c) to increase the crest height, and (d) to provide roadway access along the crest for construction or maintenance purposes.

Massive concrete caps have been used with cover layers of precast concrete armor units to replace armor units of questionable stability on an overtopped crest and to provide a rigid backup to the top rows of armor units on the slopes. To accomplish this dual purpose, the cap can be a slab with a solid or permeable parapet (Czerniak and Collins, 1977; Jensen, 1983; and Fig. 6-64, (see Ch. 6)), a slab over stone grouted to the bottom elevation of the armor layer (Figs. 6-60 and 6-63, or a solid or permeable block (Lillevang, 1977, Markle, 1982, and Fig. 6-65)).

Concrete caps with solid vertical or sloped walls reflect waves out through the upper rows of armor units, perhaps causing loss of those units. Solid slabs and blocks can trap air beneath them, creating uplift forces during heavy wave action that may crack or tip the cap (Magoon, Sloan, and Foote, 1974). A permeable cap decreases both of these problems. A parapet can be made permeable, and vertical vents can be placed through the slab or block itself (Mettam, 1976).

Lillevang (1977) designed a breakwater crest composed of a vented block cap placed on an unchinked, ungrouted extension of the seaward slope's under-layer, a permeable base reaching across the crest. Massive concrete caps must be placed after a structure has settled or must be sufficiently flexible to undergo settlement without breaking up (Magoon, Sloan, and Foote, 1974).

Ribbed caps are a compromise between the solid block and a covering of concrete armor units. The ribs are large, long, rectangular members of reinforced concrete placed perpendicular to the axis of a structure in a manner resembling railroad ties. The ribs are connected by reinforced concrete braces, giving the cap the appearance of a railroad track running along the structure crest. This cap serves to brace the upper units on the slopes, yet is permeable in both the horizontal and vertical directions. Ribbed caps have been used on Corps breakwaters at Maalea Harbor (Carver and Markle, 1981a), at Kahului (Markle, 1982), on Maui, and at Pohoiki Bay, all in the State of Hawaii.

Waves overtopping a concrete cap can damage the leeside armor layer (Magoon, Sloan, and Foote, 1974). The width of the cap and the shape of its lee side can be designed to deflect overtopping waves away from the structure's lee side (Czerniak and Collins, 1977; Lillevang, 1977; and Jensen, 1983). Ribbed caps help dissipate waves.

High parapet walls have been added to caps to deflect overtopping seaward and allow the lowering of the crest of the rubble mound itself. These walls present the same reflection problems described above and complicate the design.
of a stable cap (Mettam, 1976; Jensen, 1983). Hydraulic model tests by Carver and Davidson (1976; 1983) have investigated the stability of caps with high parapet walls proposed for Corps structures.

To evaluate the need for a massive concrete cap to increase structural stability against overtopping, consideration should be given to the cost of including a cap versus the cost of increasing dimensions (a) to prevent overtopping and (b) for construction and maintenance purposes. A massive concrete cap is not necessary for the structural stability of a structure composed of concrete armor units when the difference in elevation between the crest and the limit of wave runup on the projected slope above the structure is less than 15 percent of the total wave runup. For this purpose, an all-rubble structure is preferable, and a concrete cap should be used only if substantial savings would result. Maintenance costs for an adequately designed rubble structure are likely to be lower than for any alternative composite-type structure.

The cost of a concrete cap should also be compared to the cost of covering the crest with flexible, permeable concrete armor units, perhaps larger than those used on the slopes, or large quarystone armor. Bottin, Chatham, and Carver (1976) conducted model tests on an overtopped breakwater with dolos armor on the seaward slope, but with large quarystone on the crest. The breakwater at Pria, Terceria, Azores, was repaired using large quarystone instead of a concrete cap on the crest to support the primary tetrapod armor units. Two rows of large armor stones were placed along the shoreward side of the crest to stabilize the top row of tetrapods. An inspection in March 1970 indicated that this placement has performed satisfactorily even though the structure has been subjected to wave overtopping.

Hydraulic model tests are recommended to determine the most stable and economical crest designs for major structures.

Experience indicates that concrete placed in the voids on the structure slopes has little structural value. By reducing slope roughness and surface porosity, the concrete increases wave runup. The effective life of the concrete is short, because the bond between concrete and stone is quickly broken by structure settlement. Such filling increases maintenance costs. For a roadway, a concrete cap can usually be justified if frequent maintenance of armored slopes is anticipated. A smooth surface is required for wheeled vehicles; tracked equipment can be used on ribbed caps.

(3) Thickness of Armor Layer and Underlayers and Number of Armor Units. The thickness of the cover and underlayers and the number of armor units required can be determined from the following formulas:

\[
 r = n \, k \Delta \left( \frac{W}{w_r} \right)^{1/3}
\]

where \( r \) is the average layer thickness in meters (or feet), \( n \) is the number of quarystone or concrete armor units in thickness comprising the cover layer, \( W \) is the mass of individual armor units in kilograms (or weight in pounds), and \( w_r \) is the mass density in kilograms per cubic meter (or unit weight in pounds per cubic foot). The placing density is given by

\[
7-236
\]
\[
\frac{N_r}{A} = n \ k_\Delta \left( 1 - \frac{P}{100} \right) \left( \frac{w_r}{w} \right)^{2/3}
\] 

(7-122)

where \( N_r \) is the required number of individual armor units for a given surface area, \( A \) is surface area, \( k_\Delta \) is the layer coefficient, and \( P \) is the average porosity of the cover layer in percent. Values of \( k_\Delta \) and \( P \), determined experimentally, are presented in Table 7-13.

The thickness \( r \) of a layer of riprap is either 0.30 m, or one of the following:

\[
r = 2.0 \left( \frac{w_{50}}{w_r} \right)^{1/3}
\] 

(7-123)

where \( w_{50} \) is the weight of the 50 percent size in the gradation, or

\[
r = 1.25 \left( \frac{w_{\text{max}}}{w_r} \right)^{1/3}
\] 

(7-124)

where \( w_{\text{max}} \) is the heaviest stone in the gradation, whichever of the three is the greatest. The specified layer thickness should be increased by 50 percent for riprap placed underwater if conditions make placement to design dimensions difficult. The placing density of riprap is calculated as the weight of stone placed per unit area of structure slope, based on the measured weight per unit volume of riprap. The placing density may be estimated as the product of the layer thickness \( r \), the unit weight of the rock \( w_r \), and \( \left( 1 - \frac{P}{100} \right) \).

(4). Bottom Elevation of Primary Cover Layer. The armor units in the cover layer (the weights are obtained by eq. 7-116) should be extended downslope to an elevation below minimum SWL equal to the design wave height \( H \) when the structure is in a depth \( > 1.5H \), as shown in Figure 7-116. When the structure is in a depth \( < 1.5H \), armor units should be extended to the bottom, as shown in Figure 7-117.

On revetments located in shallow water, the primary cover layer should be extended seaward of the structure toe on the natural bottom slope as scour protection.

The larger values of \( K_D \) for special-placement parallelepiped stone in Table 7-8 can be obtained only if a toe mound is carefully placed to support the quarrystones with their long axes perpendicular to the structure slope (U.S. Army Corps of Engineers, 1979). For dolosse, it is recommended that the bottom rows of units in the primary cover layer be "special placed" on top of the secondary cover layer (Fig. 7-116), the toe berm (Fig. 7-117), or the bottom itself, whenever wave conditions and water clarity permit. Site-specific model studies have been performed with the bottom units placed with their vertical flukes away from the slope and the second row of dolosse placed on or overtopping the horizontal flukes of the lower units to assure that the units interlock with the random-placed units farther up the slope (Carver, 1976;
Bottin, Chatham, and Carver, 1976). The tests indicated that special placement of the bottom dolosse produces better toe stability than random placement. The seaward dolosse in the bottom row should be placed with the bottom of the vertical flukes one-half the length of the units (dimension C in Fig. 7-112) back from the design surface of the primary armor layer to produce the design layer thickness. Model tests to determine the bottom elevation of the primary cover layer and the type of armor placement should be made whenever economically feasible.

(5) Toe Berm for Cover Layer Stability. As illustrated in Figure 7-117, structures exposed to breaking waves should have their primary cover layers supported by a quarystone toe berm. For preliminary design purposes the quarystone in the toe berm should weigh W/10, where W is the weight of quarystone required for the primary cover layer as calculated by equation (7-116) for site conditions. The toe berm stone can be sized in relation to W even if concrete units are used as primary armor. The width of the top of the berm is calculated using equation (7-120), with n = 3. The minimum height of the berm is calculated using equation (7-121), with n = 2.

Model tests can establish whether the stone size or berm dimensions should be varied for the final design. Tests may show an advantage to adding a toe berm to a structure exposed to nonbreaking waves.

The toe berm may be placed before or after the adjacent cover layer. It must be placed first, as a base, when used with special-placement quarystone or uniform-placement tribars. When placed after the cover layer, the toe berm must be high enough to provide bracing up to at least half the height of the toe armor units. The dimensions recommended above will exceed this requirement.

(6) Structure Head and Lee Side Cover Layer. Armoring of the head of a breakwater or jetty should be the same on the lee side slope as on the seaside slope for a distance of about 15 to 45 meters from the structure end. This distance depends on such factors as structure length and crest elevation at the seaward end.

Design of the lee side cover layer is based on the extent of wave overtopping, waves and surges acting directly on the lee slope, porosity of the structure, and differential hydrostatic head resulting in uplift forces which tend to dislodge the back slope armor units.

If the crest elevation is established to prevent possible overtopping, the weight of armor units and the bottom elevation of the back slope cover layer should theoretically depend on the lesser wave action on the lee side and the porosity of the structure. When minor overtopping is anticipated, the armor weight calculated for the seaward side primary cover layer should be used on the lee side, at least down to the SWL or -0.5 H for preliminary design; however, model testing may be required to establish an armor weight stable under overtopping wave impact. Primary armor on the lee side should be carried to the bottom for breakwaters with heavy overtopping in shallow water (breaking wave conditions), as shown in Figure 7-117. Equation 7-116 cannot be used with values of $K_D$ listed in Table 7-8 calculate leeside armor weight under overtopping, since the $K_D$ values were established for armor on
the seaward side and may be incorrect for leeside concrete or quarrystone units (Merrifield, 1977; Lillevang, 1977). The presence of a concrete cap will also affect overtopping forces on the lee side in ways that must be quantified by modeling. When both side slopes receive similar wave action (as with groins or jetties), both sides should be of similar design.

(7) **Secondary Cover Layer.** If the armor units in the primary and secondary cover layers are of the same material, the weight of armor units in the secondary cover layer, between -1.5 H and -2.0 H, should be greater than about one-half the weight of armor units in the primary cover layer. Below -2.0 H, the weight requirements can be reduced to about W/15 for the same slope condition (see Fig. 7-116). If the primary cover layer is of quarrystone, the weights for the secondary quarrystone layers should be ratioed from the weight of quarrystone that would be required for the primary cover layer. The use of a single size of concrete armor for all cover layers—i.e., upgrading the secondary cover layer to the same size as the primary cover layer—may prove to be economically advantageous when the structure is located in shallow water (Fig. 7-117); in other words, with depth \( d \leq 1.5 \ H \), armor units in the primary cover layer should be extended down the entire slope.

The secondary cover layer (Fig. 7-116) from -1.5 H to the bottom should be as thick as or thicker than the primary cover layer. For cover layers of quarrystone, for example, and for the preceding ratios between the armor weight \( W \) in the primary cover layer and the quarrystone weight in the secondary cover layers, this means that if \( n = 2 \) for the primary cover layer (two quarrystones thick) then \( n = 2.5 \) for the secondary cover layer from -H to -2.0 H and \( n = 5 \) for that part of the secondary cover layer below -2.0 H.

The interfaces between the secondary cover layers and the primary cover layer are shown at the slope of 1-on-1.5 in Figure 7-116. Steeper slopes for the interfaces may contribute to the stability of the cover armor, but material characteristics and site wave conditions during construction may require using a flatter slope than that shown.

(8) **Underlayers.** The first underlayer directly beneath the primary armor units should have a minimum thickness of two quarrystones \( (n = 2) \) (see Figs. 7-116 and 7-117). For preliminary design these should weigh about one-tenth the weight of the overlying armor units \( (W/10) \) if (a) the cover layer and first underlayer are both quarrystone, or (b) the first underlayer is quarrystone and the cover layer is concrete armor units with a stability coefficient \( K_D \leq 12 \) (where \( K_D \) is for units on a trunk exposed to nonbreaking waves). When the cover layer is of armor units with \( K_D > 12 \) such as dolosse, toskanes, and tribars (placed uniformly in a single layer), the first underlayer quarrystone weight should be about \( W/5 \) or one-fifth the weight of the overlying armor units. The larger size is recommended to increase interlocking between the first underlayer and the armor units of high \( K_D \). Carver and Davidson (1977) and Carver (1980) found, from hydraulic model tests of quarrystone armor units and dolosse placed on a breakwater trunk exposed to nonbreaking waves, that the underlayer stone size could range from \( W/5 \) to \( W/20 \), with little effect on stability, runup, or rundown. If the underlayer stone proposed for a given structure is available in weights from \( W/5 \) to \( W/20 \), the structure should be model tested with a first underlayer of the available stone before the design is made final. The tests
will determine whether this economical material will support a stable primary cover layer of the planned armor units when exposed to the site conditions.

The second underlayer beneath the primary cover layer and upper secondary cover layer (above -2.0 H) should have a minimum equivalent thickness of two quarystones; these should weigh about one-twentieth the weight of the immediately overlying quarystones \((1/20 \times W/10 = W/200)\) for quarystone and some concrete primary armor units.

The first underlayer beneath the lower secondary cover layer (below -2.0 H), should also have a minimum of two thicknesses of quarystone (see Fig. 7-116); these should weigh about one-twentieth of the immediately overlying armor unit weight \((1/20 \times W/15 = W/300)\) for units of the same material. The second underlayer for the secondary armor below -2.0 H can be as light as \(W/6,000\), or equal to the core material size.

Note in the "recommended" section of Figure 7-116 that when the primary armor is quarystone and/or concrete units with \(K_D \leq 12\), the first underlayer and second (below -2.0 H) quarystone sizes are \(W/10\) to \(W/15\). If the primary armor is concrete armor units with \(K_D > 12\), the first underlayer and secondary (below -2.0 H) quarystone sizes are \(W/5\) and \(W/10\).

For a graded riprap cover layer, the minimum requirement for the under-layers, if one or more are necessary, is

\[
D_{15} \text{(cover)} \leq 5D_{85} \text{(under)}
\]

where \(D_{15}\) (cover) is the diameter exceeded by the coarsest 85 percent of the riprap or underlayer on top and \(D_{85}\) (under) is the diameter exceeded by the coarsest 15 percent of the underlayer or soil below (Ahrens, 1981). For a revetment, if the riprap and the underlying soil satisfy the size criterion, no underlayer is necessary; otherwise, one or more are required. The size criterion for riprap is more restrictive than the general filter criterion given at the beginning of Section III,7,g, above, and repeated below. The riprap criterion requires larger stone in the lower layer to prevent the material from washing through the voids in the upper layer as its stones shift under wave action. A more conservative underlayer than that required by the minimum criterion may be constructed of stone with a 50 percent size of about \(W50/20\). This larger stone will produce a more permeable underlayer, perhaps reducing runup, and may increase the interlocking between the cover layer and underlayer; but its gradation must be checked against that of the underlying soil in accordance with the criterion given above.

The underlayers should be at least three 50 percent-size stones thick, but not less than 0.23 meter (Ahrens, 1981). The thickness can be calculated using equation (7-123) with a coefficient of 3 rather than 2. Note that, since a revetment is placed directly on the soil or fill of the bank it protects, a single underlayer also functions as a bedding layer or filter blanket.

(9) Filter Blanket or Bedding Layer. Foundation conditions for marine structures require thorough evaluation. Wave action against a rubble
structure, even at depths usually considered unaffected by such action, creates turbulence within both the structure and the underlying soil that may draw the soil into the structure, allowing the rubble itself to sink. Revetments and seawalls placed on sloping beaches and banks must withstand groundwater pressure tending to wash underlying soil through the structure. When large quarry stones are placed directly on a sand foundation at depths where waves and currents act on the bottom (as in the surf zone), the rubble will settle into the sand until it reaches the depth below which the sand will not be disturbed by the currents. Large amounts of rubble may be required to allow for the loss of rubble because of settlement. This settlement, in turn, can provide a stable foundation; but a rubble structure can be protected from excessive settlement resulting from leaching, undermining, or scour, by the use of either a filter blanket or bedding layer.

It is advisable to use a filter blanket or bedding layer to protect the foundations of rubble-mound structures from undermining except (a) where depths are greater than about three times the maximum wave height, (b) where the anticipated current velocities are too weak to move the average size of foundation material, or (c) where the foundation is a hard, durable material (such as bedrock).

When the rubble structure is founded on cohesionless soil, especially sand, a filter blanket should be provided to prevent differential wave pressures, currents, and groundwater flow from creating a quick condition in the foundation by removing sand through voids of the rubble and thus causing settlement. A filter blanket under a revetment may have to retain the foundation soil while passing large volumes of groundwater. Foundations of coarse gravel may be too heavy and permeable to produce a quick condition, while cohesive foundation material may be too impermeable.

A foundation that does not require a filter blanket may require a protective bedding layer. A bedding layer prevents erosion during and after construction by dissipating forces from horizontal wave, tide, and longshore currents. It also acts as a bearing layer that spreads the load of overlying stone (a) on the foundation soil to prevent excessive or differential settlement, and (b) on the filter material to prevent puncture. It interlocks with the overlying stone, increasing structure stability on slopes and near the toe. In many cases a filter blanket is required to hold foundation soil in place but a bedding layer is required to hold the filter in place. Gradation requirements of a filter layer depend principally on the size characteristics of the foundation material. If the criterion for filter design (Sowers and Sowers, 1970) is used, $D_{15}$ (filter) is less than or equal to $5D_{85}$ (foundation) (i.e., the diameter exceeded by the coarsest 85 percent of the filter material must be less than or equal to 5 times the diameter exceeded by the coarsest 15 percent of the foundation material) to ensure that the pores in the filter are too small to allow passage of the soil. Depending on the weight of the quarry stone in the structure, a geotextile filter may be used (a) instead of a mineral blanket, or (b) with a thinner mineral blanket. Geotextiles are discussed in Chapter 6 and by Moffatt and Nichol, Engineers (1983) and Eckert and Callender (1984), who present detailed requirements for using geotextile filters beneath quarry stone armor in coastal structures. A geotextile, coarse gravel, or crushed stone filter may be
placed directly over a sand, but silty and clayey soils and some fine sands must be covered by a coarser sand first. A bedding layer may consist of quarry spalls or other crushed stone, of gravel, or of stone-filled gabions. Quarry spalls, ranging in size from 0.45 to 23 kilograms, will generally suffice if placed over a geotextile or coarse gravel (or crushed stone) filter meeting the stated filter design criteria for the foundation soil. Bedding materials must be placed with care on geotextiles to prevent damage to the fabric from the bedding materials, as well as from heavier materials placed above.

Filter blanket or bedding layer thickness depends generally on the depth of water in which the material is to be placed and the size of quarystone used, but should not be less than 0.3 meter to ensure that bottom irregularities are completely covered. A filter blanket or bedding layer may be required only beneath the bottom edge of the cover and underlayers if the core material will not settle into or allow erosion of foundation material. Core material that is considerably coarser than the underlying foundation soil may need to be placed on a blanket or layer as protection against scour and settlement. It is also common practice to extend the bedding layer at least 1.5 meters beyond the toe of the cover stone. Details of typical rubble structures are shown in Chapter 6, STRUCTURAL FEATURES. In low rubble-mound structures composed entirely of cover and underlayers, leaving no room for a core, the bedding layer is extended across the full width of the structure. Examples are low and submerged breakwaters intended to control sand transport by dissipating waves (Markle and Carver, 1977) and small breakwaters for harbor protection (Carver and Markle, 1981b).


Forces of waves on rubble structures have been studied by several investigators (see Section 7, above). Brebner and Donnelly (1962) studied stability criteria for random-placed rubble of uniform shape and size used as foundation and toe protection at vertical-faced, composite structures. In their experiments, the shape and size of the rubble units were uniform, that is, subrounded to subangular beach gravel of 2.65 specific gravity. In practice, the rubble foundation and toe protection would be constructed with a core of dumped quarry-run material. The superstructure might consist of concrete or timber cribs founded on the core material or a pair of parallel-tied walls of steel sheet piling driven into the rubble core. Finally, the apron and side slope of the core should be protected from erosion by a cover layer of armor units (see Sec. d and e below).

a. Design Wave Heights. For a composite breakwater with a superstructure resting directly on a rubble-mound foundation, structural integrity may depend on the ability of the foundation to resist the erosive scour by the highest waves. Therefore, it is suggested that the selected design wave height \( H \) for such structures be based on the following:

(1) For critical structures at open exposed sites where failure would be disastrous, and in the absence of reliable wave records, the design wave height \( H \) should be the average height of the highest 1 percent of all waves \( H_1 \) expected during an extreme event, based on the deepwater significant wave height \( H_o \) corrected for refraction and shoaling. (Early
breaking might prevent the 1 percent wave from reaching the structure; if so, the maximum wave that could reach the structure should be taken for the design value of $H_1$.

(2) For less critical structures, where some risk of exceeding design assumptions is allowable, wave heights between $H_{10}$ and $H_1$ are acceptable.

The design wave for rubble toe protection is also between $H_{10}$ and $H_1$.

b. **Stability Number.** The stability number ($N_s$) is primarily affected by the depth of the rubble foundation and toe protection below the still-water level $d_1$ and by the water depth at the structure site, $d_s$. The relation between the depth ratio $d_1/d_s$ and $N_s^3$ is indicated in Figure 7-120. The cube value of the stability number has been used in the figure to facilitate its substitution in equation (7-125).

c. **Armor Stone.** The equation used to determine the armor stone weight is a form of equation (7-116):

\[
W = \frac{w_r H^3}{N_s^3 (S_r - 1)^3}
\]

(7-125)

where

- $W$ = mean weight of individual armor unit, newtons or pounds.
- $w_r$ = unit weight of rock (saturated surface dry), newtons per cubic meter or pounds per cubic foot (Note: substitution of $\rho_r$, the mass density of the armor material in kilograms per cubic meter or slugs per cubic foot, will yield $W$ in units of mass (kilograms or slugs)
- $H$ = design wave height (i.e., the incident wave height causing no damage to the structure)
- $S_r$ = specific gravity of rubble or armor stone relative to the water on which the structure is situated ($S_r = w_r / w_w$)
- $w_w$ = unit weight of water, fresh water = 9,800 newtons per cubic meter (62.4 pounds per cubic foot), seawater = 10,047 newtons per cubic meter (64.0 pounds per cubic foot). (Note: substitution of $\left(\frac{\rho_r - \rho_w}{\rho_w}\right)^3$, where $\rho_w$ is the mass density of the water at the structure, for $(S_r - 1)^3$ yields the same result.)
- $N_s$ = design stability number for rubble foundations and toe protection (see Fig. 7-120).
Figure 7-120. Stability number $N_s$ for rubble foundation and toe protection.

\[
W = \frac{w_r H^3}{N_s^3 (S_r - 1)^3}
\]

and $B = 0.4 d_s$
d. Scour Protection. The forces causing loss of foundation soil from beneath a rubble-mound structure are accentuated at the structure toe. Wave pressure differentials and groundwater flow may produce a quick condition at the toe, then currents may carry the suspended soil away. A shallow scour hole may remove support for the cover layers, allowing them to slump down the face, while a deep hole may destabilize the slope of the structure, over-steepening it until bearing failure in the foundation soil allows the whole face to slip. Toe protection in the form of an apron must prevent such damage while remaining in place under wave and current forces and conforming to an uneven bottom that may be changing as erosion occurs.

Toe scour is a complex process. The toe apron width and stone size required to prevent it are related to the wave and current intensity; the bottom material; and the slope, roughness, and shape of the structure face. No definitive method for designing toe protection is known, but some general guidelines for planning toe protection are given below. The guidelines will provide only approximate quantities which may require doubling to be conservative, in some cases. A detailed study of scour in the natural bottom and near existing structures should be conducted at a planned site, and model studies should be considered before determining a final design.

(1) Minimum Design. Hales (1980) surveyed scour protection practices in the United States and found that the minimum toe apron was an extension of the bedding layer and any accompanying filter blanket measuring 0.6 to 1.0 meter thick and 1.5 meters wide. In the northwest United States, including Alaska, aprons are commonly 1.0 to 1.5 meters thick and 3.0 to 7.5 meters wide. Materials used, for example, were bedding of quarry-run stone up to 0.3 meter in dimension or of gabions 0.3 meter thick; core stone was used if larger than the bedding and required for stability against wave and current forces at the toe.

(2) Design for Maximum Scour Force. The maximum scour force occurs where wave downrush on the structure face extends to the toe. Based on Eckert (1983), the minimum toe apron will be inadequate protection against wave scour if the following two conditions hold. The first is the occurrence of water depth at the toe that is less than twice the height of the maximum expected unbroken wave that can exist in that water depth. The maximum unbroken wave is discussed in Chapter 5 and is calculated using the maximum significant wave height $H_{\text{sm}}$ from Figure 3-21, and methods described in Section I of this chapter. Available wave data can be used to determine which calculated wave heights can actually be expected for different water levels at the site.

The second condition that precludes the use of a minimum toe apron is a structure wave reflection coefficient $\chi$ that equals or exceeds 0.25, which is generally true for slopes steeper than about 1 on 3. If the reflection coefficient is lower than the limit, much of the wave force will be dissipated on the structure face and the minimum apron width may be adequate. If the toe apron is exposed above the water, especially if waves break directly on it, the minimum quarrystone weight will be inadequate, whatever the slope.

(3) Tested Designs. Movable bed model tests of toe scour protection for a quarrystone-armored jetty with a slope of 1 on 1.25 were performed by Lee (1970; 1972). The tests demonstrated that a layer two stones thick of
stone weighing about one-thirtieth the weight of primary cover layer armor (W/30) was stable as cover for a core-stone apron in water depths of more than one but less than two wave heights. The width of the tested aprons was four to six of the aprons cover layer stones, and so could be calculated using equation (7-120) with \( n = 4 \) to 6 and \( W = w r / 30 \).

Hales (1980) describes jetties, small breakwaters, and revetments with slopes of 1 on 3 or steeper and toes exposed to intense wave action in shallow water that have their aprons protected by a one-stone-thick layer of primary cover layer quarystone. The aprons were at least three to four cover stones wide; i.e., if equation (7-120) were used, \( n = 3 \) to 4 and \( W = w r \). In Hawaii, the sediment beneath the toes of such structures was excavated down to coral; or, if the sand was too deep, the toe apron was placed in a trench 0.6 to 2.0 meters deep.

(4) **Materials.** The quarystone of the structure underlayers, secondary cover layer, toe mound for cover layer stability, or the primary cover layer itself can be extended over a toe apron as protection, the size of which depends on the water depth, toe apron thickness, and wave height. Eckert (1983) recommended that, in the absence of better guidance, the weight of cover for a submerged toe exposed to waves in shallow water be chosen using the curve in Figure 7-120 for a rubble-mound foundation beneath a vertical structure and equation (7-125) as a guide. The design wave height \( H \) to be used in equation (7-125) is the maximum expected unbroken wave that occurs at the structure during an extreme event, and the design water depth is the minimum that occurs with the design wave height. Since scour aprons generally are placed on very flat slopes, quarystone of the size in an upper secondary cover layer \( w r / 2 \) probably will be the heaviest required unless the apron is exposed above the water surface during wave action. Quarystone of primary cover layer size may be extended over the toe apron if the stone will be exposed in the troughs of waves, especially breaking waves. The minimum thickness of cover over the toe apron should be two quarystones, unless primary cover layer stone is used.

(5) **Shallow-Water Structures.** The width of the apron for shallow-water structures with reflection coefficients equalling or greater than 0.25 can be planned from the structure slope and the expected scour depth. As discussed in Chapter 5, the maximum depth of a scour trough due to wave action below the natural bed is about equal to the maximum expected unbroken wave at the site. To protect the stability of the face, the toe soil must be kept in place beneath a surface defined by an extension of the face surface into the bottom to the maximum depth of scour. This can be accomplished by burying the toe, where construction conditions permit, thereby extending the face into an excavated trench the depth of the expected scour. Where an apron must be placed on the existing bottom or only can be partially buried, its width can be made equal to that of a buried toe; i.e., equal to the product of the expected scour depth and the cotangent of the face slope angle.

(6) **Current Scour.** Toe protection against currents may require smaller protective stone, but wider aprons. Stone size can be estimated from Section IV below. The current velocity used for selecting stone size, the scour depth to be expected, and the resulting toe apron width required can be
estimated from site hydrography, measured current velocities, and model studies (Hudson et al., 1979). Special attention must be given to sections of the structure where scour is intensified; i.e. to the head, areas of a section change in alinement, bar crossings, the channel sides of jetties, and the downdrift sides of groins. Where waves and currents occur together, Eckert (1983) recommends increasing the cover size by a factor of 1.5. The stone size required for a combination of wave and current scour can be used out to the width calculated for wave scour protection; smaller stone can be used beyond that point for current scour protection. Note that the conservatism of the apron width estimates depends on the accuracy of the methods used to predict the maximum depth of scour.

(7) Revetments. Revetments commonly are typically the smallest and most lightly armored of coastal protective structures, yet their failure leads directly to loss of property and can put protected structures in jeopardy. They commonly are constructed above the design water level or in very shallow water where their toes are likely to be exposed to intense wave and current forces during storms. For these reasons, their toes warrant special protection.

Based on guidance in EM 1110-2-1614 (U.S. Army Corps of Engineers, 1984), the cover for the toe apron of a revetment exposed to waves in shallow water should be an extension of the lowest cover layer on the revetment slope. Only the cover thickness is varied to increase stability. The toe apron should be buried wherever possible, with the revetment cover layer extended into the bottom for at least the distance of 1 meter or the maximum expected unbroken wave height, whichever is greater. If scour activity is light, the thickness of the cover on the buried toe can be a minimum of two armor stones or 50 percent size stones in a riprap gradation, the same as on the slope. For more intense scour, the cover thickness should be doubled and the extension depth increased by a factor of up to 1.5. For the most severe scour, the buried toe should be extended horizontally an additional distance equal to twice the toe's depth, that is, 2 to 3 times the design wave height (see Fig. 7-121).

If the apron is a berm placed on the existing bottom and the cover is quarystone armor, the cover thickness may be as little as one stone and the apron width may be three to four stones. A thickness of two stones and a width equal to that of a buried toe is more conservative and recommended for a berm covered by riprap. For the most severe wave scour the thickness should be doubled and a width equal to 3 to 4.5 design wave heights used, as illustrated in Figure 7-121. According to EM 1110-2-1601 (U.S. Army Corps of Engineers, 1970), the width of a toe apron exposed to severe current scour should be five times the thickness of the revetment cover layer, whether the toe is buried or a berm.

If a geotextile filter is used beneath the toe apron of a revetment or a structure that passes through the surf zone, such as a groin, the geotextile should not be extended to the outer edge of the apron. It should stop about a meter from the edge to protect it from being undermined. As an alternative, the geotextile may be extended beyond the edge of the apron, folded back over the bedding layer and some of the cover stone, and then buried in cover stone and sand to form a Dutch toe. This additionally stable form of toe is illustrated as an option in Figure 7-121.
If a revetment is overtopped, even by minor splash, the stability can be affected. Overtopping can (a) erode the area above or behind the revetment, negating the structure's purpose; (b) remove soil supporting the top of the revetment, leading to the unraveling of the structure from the top down; and (c) increase the volume of water in the soil beneath the structure, contributing to drainage problems. The effects of overtopping can be limited by choosing a design height exceeding the expected runup height or by armoring the bank above or behind the revetment with a splash apron. The splash apron
can be a filter blanket covered by a bedding layer and, if necessary to prevent scour by splash, quarrystone armor or riprap; i.e., an apron similar in design to a toe apron. The apron can also be a pavement of concrete or asphalt which serves to divert overtopping water away from the revetment, decreasing the volume of groundwater beneath the structure.

e. Toe Berm for Foundation Stability. Once the geometry and material weights of a structure are known, the structure's bearing pressure on the underlying soil can be calculated. Structure settlement can be predicted using this information, and the structure's stability against a slip failure through the underlying soil can be analyzed (Eckert and Callender, 1984). If a bearing failure is considered possible, a quarrystone toe berm sufficiently heavy to prevent slippage can be built within the limit of the slip circle. This berm can be combined with the toe berm supporting the cover layer and the scour apron into one toe construction.

If the vertical structure being protected by a toe berm is a cantilevered or anchored sheet-pile bulkhead, the width of the berm \( B \) must be sufficient to cover the zone of passive earth support in front of the wall. Eckert and Callender (1984) describe methods of determining the width of this zone. As an approximation, \( B \) should be the greatest of (a) twice the depth of pile penetration, (b) twice the design wave height, or (c) \( 0.4 \, d_g \) (Eckert, 1983). If the vertical structure is a gravity retaining wall, the width of the zone to be protected can be estimated as the wall height, the design wave height, or \( 0.4 \, d_g \), whichever is greatest.

IV. VELOCITY FORCES—STABILITY OF CHANNEL REVETMENTS

In the design of channel revetments, the armor stone along the channel slope should be able to withstand anticipated current velocities without being displaced (Cox, 1958; Cambell, 1966).

The design armor weight is chosen by calculating the local boundary shear expected to act on a revetment and the shear that a design stone weight can withstand. Since the local boundary shear is a function of the revetment surface roughness, and the roughness is a function of the stone size, a range of stone sizes must be evaluated until a size is found which is stable under the shear it produces.

When velocities near the revetment boundaries are available from model tests, prototype measurements, or other means, the local boundary shear is

\[
\tau_b = \frac{\omega_b}{g} \left( \frac{V}{5.75 \log_{10} \left( \frac{30y}{d_g} \right)} \right)^2
\]

where

\( \tau_b \) = local boundary shear

\( V \) = the velocity at a distance \( y \) above the boundary

\( d_g \) = equivalentarmor unit diameter; i.e.,

7-249
The maximum velocity of tidal currents in midchannel through a navigation opening as given by Sverdrup, Johnson, and Fleming (1942) can be approximated by

\[ V = \frac{4\pi Ah}{3TS} \]  

(7-128)

where

\[ V \] = maximum velocity at center of opening

\[ T \] = period of tide

\[ A \] = surface area of harbor

\[ S \] = cross section area of openings

\[ h \] = tidal range

The current velocity at the sides of the channel is about two-thirds the velocity at midchannel; therefore, the velocity against the revetments at the sides can be approximated by

\[ V = \frac{8}{9} \frac{\pi Ah}{3TS} \]  

(7-129)

If no prototype or model current velocities are available, this velocity can be used as an approximation of \( V \) and to calculate the local boundary shear.

If the channel has a uniform cross section with identical bed and bank armor materials, on a constant bottom slope over a sufficient distance to produce uniform channel flow at normal depth and velocity, velocity can be calculated using the procedures described in Appendix IV of EM 1110-2-1601 (Office, Chief of Engineers, U.S. Army, 1970), or Hydraulic Design Charts available from the U.S. Army Engineer Waterways Experiment Station, Vicksburg, Miss.). In tidal channels, different water surface elevations at the ends of the channel are used to find the water surface elevation difference that gives the maximum flow volume and flow velocity. If the conditions described above hold, such that the flow if fully rough and the vertical velocity distribution is logarithmic, the local boundary shear \( \tau_b \) is

\[ \tau_b = \frac{w_r}{g} \left( \frac{V}{5.75 \log_{10} \left( \frac{12.1}{d} \right)} \right)^2 \]  

(7-130)
where

\( \bar{V} \) = average local velocity in the vertical

\( d \) = depth at site (\( \bar{V} \) is average over this depth)

If the channel is curved, the computed local boundary shear should be multiplied by a factor appropriate for that cross section (available in EM 1110-2-1601, Office, Chief of Engineers, 1970). If the conditions described above leading to a uniform channel flow at normal depth and velocity do not exist, as they will not for most tidal channels, the local boundary shear computed from the equation above should be increased by a factor of 1.5.

If the local boundary shear can be calculated by using the average velocity over depth, it should also be calculated using an estimated velocity at the revetment surface, as described in the two methods above. The calculated local boundary shears can be compared and the most conservative used.

Calculate the riprap design shear or armor stone design shear using

\[
\tau = 0.040 (w_r - W_r) d_g
\]  

(7-131)

where \( \tau \) = design shear for the channel bottom if essentially level, and

\[
\tau' = \tau \left(1 - \frac{\sin^2 \theta}{\sin^2 \phi}\right)^{1/2}
\]  

(7-132)

where

\( \tau' \) = design shear for channel side slopes

\( \theta \) = angle of side slope with horizontal

\( \phi \) = angle of repose of the riprap (normally about 40°)

For all graded stone armor (riprap), the gradation should have the following relations to the computed value for \( W_{50\ min} \):

\[
W_{100\ max} = 5 W_{50\ min}
\]  

(7-133)

\[
W_{100\ min} = 2 W_{50\ min}
\]  

(7-134)

\[
W_{50\ max} = 1.5 W_{50\ min}
\]  

(7-135)

\[
W_{15\ max} = 0.5 W_{50\ max}
\]  

(7-136)
\[ W_{15 \text{ max}} = 0.75 W_{50 \text{ min}} \quad (7-137) \]
\[ W_{15 \text{ min}} = 0.31 W_{50 \text{ min}} \quad (7-138) \]

If stone is placed above water, the layer thickness is
\[ r = 2.1 \left( \frac{W_{50 \text{ min}}}{w_p} \right)^{1/3}, \text{ or } 0.3 \text{ m (12 in.) minimum} \quad (7-139) \]

If stone is placed below water,
\[ r = 3.2 \left( \frac{W_{50 \text{ min}}}{w_p} \right)^{1/3}, \text{ or } 0.5 \text{ m (18 in.) minimum} \quad (7-140) \]

to account for inaccuracy in placement.

Equations (7-133) through (7-138) are used by choosing a layer thickness for a type of placement, then calculating the \( d_g \) for \( W_{50 \text{ min}} \) (\( d_g \text{ min} \)) and for \( W_{50 \text{ max}} \) (\( d_g \text{ max} \)). The local boundary shear should be calculated using \( d_g \text{ max} \); the design shear should be calculated using \( d_g \text{ min} \). If the design shear matches or exceeds the local boundary shear, the layer thickness and stone sizes are correct.

For uniform stone, \( d_g \) is uniform so that the same value is used for calculating the local boundary and design shears. In the special case where the velocity is known within 3 meters of the surface of the revetment, the local boundary shear equation for velocities near the revetment surface can be used with \( y \) set equal to \( d_g \). This gives
\[ \tau_b = \frac{w_o}{g} \left( \frac{v}{5.75 \log_{10} 30} \right)^2 \]

Setting this equal to the armor stone design shear, and solving the result for \( V \) gives
\[ V = 5.75 (0.040)^{1/2} \log_{10} 30 g^{1/2} \left( \frac{w_p-w_o}{w_o} \right)^{1/2} \left( 1 - \frac{\sin^2 \theta}{\sin^2 \phi} \right)^{1/4} d_g^{1/2} \]
or
\[ V = 5.75 (0.020)^{1/2} \log_{10} 30 (2g)^{1/2} \left( \frac{w_p-w_o}{w_o} \right)^{1/2} \left( 1 - \frac{\sin^2 \theta}{\sin^2 \phi} \right)^{1/4} d_g^{1/2} \]

7-252
This is Isbash’s equation for stone embedded in the bottom of a sloped channel modified for stone embedded in a bank with angle \( \theta \) to the horizontal (the coefficient 1.20 is Isbash's constant for embedded stone). From this, the armor stone weight required to withstand the velocity \( V \) is as follows:

\[
V = 1.20 \left( 2g \right)^{1/2} \left( \frac{w_r - w_o}{w_o} \right)^{1/2} \left( \frac{1 - \sin^2 \theta}{\sin^2 \phi} \right)^{1/4} \frac{d}{g}^{1/2} \tag{7-141}
\]

\[
W = \frac{\pi}{6} \frac{v^6}{(1.20)^6 (2g)^3} \left( \frac{w_r - w_o}{w_o} \right)^3 \left( 1 - \frac{\sin^2 \theta}{\sin^2 \phi} \right)^{3/2} \tag{7-142}
\]

\[
W = 0.0219 \frac{v^6}{g} \left( \frac{w_o}{w_r - w_o} \right)^3 \left( 1 - \frac{\sin^2 \theta}{\sin^2 \phi} \right)^{-3/2}
\]

V. IMPACT FORCES

Impact forces are an important design consideration for shore structures because of the increased use of thin flood walls and gated structures as part of hurricane protection barriers. High winds of a hurricane propelling small pleasure craft, barges, and floating debris can cause great impact forces on a structure. Large floating ice masses also cause large horizontal impact forces. If site and functional condition require the inclusion of impact forces in the design, other measures should be taken: either the depth of water against the face of the structure should be limited by providing a rubble-mound absorber against the face of the wall, or floating masses should be grounded by building a partially submerged structure seaward of the shore structure that will eliminate the potential hazard and need for impact design consideration.

In many areas impact hazards may not occur, but where the potential exists (as for harbor structures), impact forces should be evaluated from impulse-momentum considerations.

VI. ICE FORCES

Ice forms are classified by terms that indicate manner of formation or effects produced. Usual classifications include sheet ice, shale, slush, frazil ice, anchor ice, and agglomerate ice (Striegl, 1952; Zumberg and Wilson, 1953; Peyton, 1968).

There are many ways ice can affect marine structures. In Alaska and along the Great Lakes, great care must be exercised in predicting the different ways in which ice can exert forces on structures and restrict operations. Most situations in which ice affects marine structures are outlined in Table 7-14.
The amount of expansion of fresh water in cooling from 12.6°C (39°F) to 0°C (32°F) is 0.0132 percent; in changing from water at 0°C (32°F) to ice at 0°C, the amount of expansion is approximately 9.05 percent, or 685 times as great. A change of ice structure to denser form takes place when with a temperature lower than -22°C (-8°F), it is subjected to pressures greater than about 200 kilonewtons per square meter (30,000 pounds per square inch). Excessive pressure, with temperatures above -22°C, causes the ice to melt. With the temperature below -22°C, the change to a denser form at high pressure results in shrinkage which relieves pressure. Thus, the probable maximum pressure that can be produced by water freezing in an enclosed space is approximately 200 kilonewtons per square meter (30,000 pounds per square inch).

Designs for dams include allowances for ice pressures of as much as 657,000 to 730,000 newtons per meter (45,000 to 50,000 pounds per linear foot). The crushing strength of ice is about 2,750 kilonewtons per square meter (400 pounds per square inch). Thrust per meter for various thicknesses of ice is about 43,000 kilonewtons for 0.5 meter, 86,000 kilonewtons for 1.0 meter, etc. Structures subject to blows from floating ice should be capable of resisting 97,650 to 120,000 kilonewtons per square meter (10 to 12 tons per square foot, or 139 to 167 pounds per square inch) on the area exposed to the greatest thickness of floating ice.

Ice also expands when warmed from temperatures below freezing to a temperature of 0°C without melting. Assuming a lake surface free of snow with an average coefficient of expansion of ice between -7°C (20°F) and 0°C equaling 0.0000512 m/m°C, the total expansion of a sheet of ice a kilometer long for a rise in temperature of 10°C (50°F) would be 0.5 meter.

Normally, shore structures are subject to wave forces comparable in magnitude to the maximum probable pressure that might be developed by an ice sheet. As the maximum wave forces and ice thrust cannot occur at the same time, usually no special allowance is made for overturning stability to resist ice thrust. However, where heavy ice, either in the form of a solid ice sheet or floating ice fields may occur, adequate precautions must be taken to ensure that the structure is secure against sliding on its base. Ice breakers may be required in sheltered water where wave action does not require a heavy structure.

Floating ice fields when driven by a strong wind or current may exert great pressure on structures by piling up on them in large ice packs. This condition must be given special attention in the design of small isolated structures. However, because of the flexibility of an ice field, pressures probably are not as great as those of a solid ice sheet in a confined area.

Ice formations at times cause considerable damage on shores in local areas, but their net effects are largely beneficial. Spray from winds and waves freezes on the banks and structures along the shore, covering them with a protective layer of ice. Ice piled on shore by wind and wave action does not, in general, cause serious damage to beaches, bulkheads, or protective riprap, but provides additional protection against severe winter waves. Ice often affects impoundment of littoral drift. Updrift source material is less erodible when frozen, and windrowed ice is a barrier to shoreward-moving wave energy; therefore, the quantity of material reaching an impounding structure.
is reduced. During the winters of 1951-52, it was estimated that ice caused a reduction in rate of 
impoundment of 40 to 50 percent at the Fort Sheridan, Illinois, groin system.

Table 7-14. Effects of ice on marine structures\(^1\).

A. Direct Results of Ice Forces on Structures.
   1. Horizontal forces.
      a. Crushing ice failure of laterally moving floating ice sheets.
      b. Bending ice failure of laterally moving floating ice sheets.
      c. Impact by large floating ice masses.
      d. Plucking forces against riprap.
   2. Vertical forces.
      a. Weight at low tide of ice frozen to structural elements.
      b. Buoyant uplift at high tide of ice masses frozen to structural elements.
      c. Vertical component of ice sheet bending failure introduced by ice breakers.
      d. Diaphragm bending forces during water level change of ice sheets frozen to  
         structural elements.
      e. Forces created because of superstructure icing by ice spray.
   3. Second-order effects.
      a. Motion during thaw of ice frozen to structural elements.
      b. Expansion of entrapped water within structural elements.
      c. Jamming of rubble between structural framing members.

B. Indirect Results of Ice Forces on Structures.
   1. Impingement of floating ice sheets on moored ships.
   2. Impact forces by ships during docking which are larger than might normally be  
      expected.
   3. Abrasion and subsequent corrosion of structural elements.

C. Low-Risk but Catastrophic Considerations.
   1. Collision by a ship caught in fast-moving, ice-covered waters.
   2. Collision by extraordinarily large ice masses of very low probability of occurrence.

D. Operational Considerations.
   1. Problems of serving offshore facilities in ice-covered waters.
   2. Unusual crane loads.
   3. Difficulty in maneuvering work boats in ice-covered waters.
   4. Limits of ice cover severity during which ships can be moored to docks.
   5. Ship handling characteristics in turning basins and while docking and undocking.
   6. The extreme variability of ice conditions from year to year.
   7. The necessity of developing an ice operations manual to outline the operational limits  
      for preventing the overstressing of structures.

\(^1\)After Peyton (1968).
Some abrasion of timber or concrete structures may be caused, and individual members may be broken or bent by the weight of the ice mass. Piling has been slowly pulled by the repeated lifting effect of ice freezing to the piles, or to attached members such as wales, and then being forced upward by a rise in water stage or wave action.

VII. EARTH FORCES

Numerous texts on soil mechanics such as those by Anderson (1948), Hough (1957), and Terzaghi and Peck (1967) thoroughly discuss this subject. The forces exerted on a wall by soil backfill depend on the physical characteristics of the soil particles, the degree of soil compaction and saturation, the geometry of the soil mass, the movements of the wall caused by the action of the backfill, and the foundation deformation. In wall design, since pressures and pressure distributions are typically indeterminate because of the factors noted, approximations of their influence must be made. Guidance for problems of this nature should be sought from one of the many texts and manuals dedicated to the subject. The following material is presented as a brief introduction.

1. Active Forces.

When a mass of earth is held back by means of a retaining structure, a lateral force is exerted on the structure. If this is not effectively resisted, the earth mass will fail and a portion of it will move sideways and downward. The force exerted by the earth on the wall is called *active earth force*. Retaining walls are generally designed to allow minor rotation about the wall base to develop this active force, which is less than the at-rest force exerted if no rotation occurs. Coulomb developed the following active force equation:

\[
P_{\alpha} = \frac{wh^2}{2} \left[ \frac{\csc \theta \sin (\theta - \phi)}{\sin (\theta + \delta)} + \frac{\csc \theta \sin (\theta - \phi) \sin (\phi - i)}{\sin (\theta - i)} \right]^2
\]

(7-143)

where

- \( P_{\alpha} \) = active force per unit length, kilonewtons per meter (pounds per linear foot) of wall
- \( w \) = unit weight of soil, kilonewtons per cubic meter (pounds per linear foot) of wall
- \( h \) = height of wall or height of fill at wall if lower than wall, meters (feet)
- \( \delta \) = angle between horizontal and backslope of wall, degrees.
- \( i \) = angle of backfill surface from horizontal, degrees
- \( \phi \) = internal angle of friction of the material, degrees
\[ \delta = \text{wall friction angle, degrees} \]

These symbols are further defined in Figure 7-122. Equation (7-143) may be reduced to that given by Rankine for the special Rankine conditions where \( \delta \) is considered equal to \( i \) and \( \theta \) equal to 90 degrees (vertical wall face). When, additionally, the backfill surface is level (\( i = 0 \) degrees), the reduced equation is

\[
P_a = \frac{wh^2}{2} \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) \tag{7-144}
\]

Figure 7-123 shows that \( P_a \) from equation (7-144) is applied horizontally.

Unit weights and internal friction angles for various soils are given in Table 7-15.

The resultant force for equation (7-143) is inclined from a line perpendicular to the back of the wall by the angle of wall friction \( \delta \) (see Fig. 7-122). Values for \( \delta \) can be obtained from Table 7-16, but should not exceed the internal friction angle of the backfill material \( \phi \) and, for conservatism, should not exceed \( (3/4) \phi \) (Office, Chief of Engineers, 1961).

2. Passive Forces.

If the wall resists forces that tend to compress the soil or fill behind it, the earth must have enough internal resistance to transmit these forces. Failure to do this will result in rupture; i.e., a part of the earth will move sideways and upward away from the wall. This resistance of the earth against outside forces is called passive earth force.

The general equation for the passive force \( P_p \) is

\[
P_p = \frac{wh^2}{2} \left[ \csc \theta \sin (\theta + \phi) \right]^2 \sqrt{\frac{\sin (\theta - \delta) - \sin (\phi - \delta) \sin (\phi + i)}{\sin (\theta - i)}} \tag{7-145}
\]

It should be noted that \( P_p \) is applied below the normal to the structure slope by an angle \( -\delta \), whereas the active force is applied above the normal line by an angle \( +\delta \) (see Fig. 7-122).

For the Rankine conditions given in Section 1 above, equation (7-145) reduces to

\[
P_p = \frac{wh^2}{2} \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) \tag{7-146}
\]

Equation (7-146) is satisfactory for use with a sheet-pile structure, assuming a substantially horizontal backfill.
Table 7-15. Unit weights and internal friction angles of soils\(^1\).

<table>
<thead>
<tr>
<th>Classification</th>
<th>Unit Weight, kg/m(^3)</th>
<th>Friction Angle (\phi^\circ)</th>
<th>Consistency</th>
<th>Soil</th>
<th>Equivalent Fluid</th>
<th>Active Case</th>
<th>Passive Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granular Materials</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Uniform Materials</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Octave SAND</td>
<td>1,474 (92)</td>
<td>1,762 (110)</td>
<td>1,490 (93)</td>
<td>2,098 (131)</td>
<td>913 (57)</td>
<td>1,105 (69)</td>
<td></td>
</tr>
<tr>
<td>Clean, uniform SAND (fine or medium)</td>
<td>1,330 (83)</td>
<td>1,890 (118)</td>
<td>1,346 (84)</td>
<td>2,178 (136)</td>
<td>832 (52)</td>
<td>1,189 (73)</td>
<td></td>
</tr>
<tr>
<td>Uniform, Illogentic SILT</td>
<td>1,081 (60)</td>
<td>1,960 (118)</td>
<td>1,037 (81)</td>
<td>1,778 (136)</td>
<td>817 (51)</td>
<td>1,189 (73)</td>
<td></td>
</tr>
<tr>
<td>Well-graded Materials</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silty SAND</td>
<td>1,366 (87)</td>
<td>2,036 (127)</td>
<td>1,610 (98)</td>
<td>2,275 (142)</td>
<td>865 (56)</td>
<td>1,263 (79)</td>
<td></td>
</tr>
<tr>
<td>Clean, fine to coarse SAND</td>
<td>1,262 (65)</td>
<td>2,210 (136)</td>
<td>1,576 (86)</td>
<td>2,171 (148)</td>
<td>849 (55)</td>
<td>1,378 (80)</td>
<td></td>
</tr>
<tr>
<td>Micaeous SAND</td>
<td>1,275 (76)</td>
<td>1,925 (120)</td>
<td>1,233 (77)</td>
<td>2,210 (138)</td>
<td>749 (48)</td>
<td>1,217 (76)</td>
<td></td>
</tr>
<tr>
<td>Silty SAND and GRAVEL</td>
<td>1,422 (89)</td>
<td>2,359 (146)</td>
<td>1,442 (90)</td>
<td>2,483 (155)</td>
<td>697 (50)</td>
<td>1,474 (93)</td>
<td></td>
</tr>
<tr>
<td>Mixed Soils</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Sandy or silty CLAY</td>
<td>961 (60)</td>
<td>2,162 (135)</td>
<td>1,602 (100)</td>
<td>2,355 (147)</td>
<td>609 (38)</td>
<td>1,362 (85)</td>
<td></td>
</tr>
<tr>
<td>2. Skilly-graded silty CLAY with stones or rock fragments</td>
<td>1,346 (86)</td>
<td>2,383 (140)</td>
<td>1,684 (115)</td>
<td>2,749 (151)</td>
<td>849 (55)</td>
<td>1,426 (89)</td>
<td></td>
</tr>
<tr>
<td>3. Well-graded GRAVEL, SAND, SILT and CLAY mixture</td>
<td>1,502 (100)</td>
<td>2,371 (148)</td>
<td>2,002 (125)</td>
<td>2,499 (156)</td>
<td>993 (62)</td>
<td>1,508 (94)</td>
<td></td>
</tr>
<tr>
<td>Clay Soils</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. CLAY (30 to 50 percent clay sizes)</td>
<td>801 (50)</td>
<td>1,794 (112)</td>
<td>1,506 (94)</td>
<td>2,130 (135)</td>
<td>497 (31)</td>
<td>1,137 (71)</td>
<td></td>
</tr>
<tr>
<td>2. Colloidal CLAY (&lt;0.002 mm. 50 percent)</td>
<td>208 (13)</td>
<td>1,698 (106)</td>
<td>1,137 (71)</td>
<td>2,050 (128)</td>
<td>128 (8)</td>
<td>1,057 (66)</td>
<td></td>
</tr>
<tr>
<td>Organic Soils</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Organic SILT</td>
<td>641 (40)</td>
<td>1,762 (110)</td>
<td>1,394 (87)</td>
<td>2,098 (131)</td>
<td>400 (25)</td>
<td>1,103 (69)</td>
<td></td>
</tr>
<tr>
<td>2. Organic CLAY (30 to 50 percent clay sizes)</td>
<td>482 (30)</td>
<td>1,602 (100)</td>
<td>1,297 (81)</td>
<td>2,002 (125)</td>
<td>286 (18)</td>
<td>993 (62)</td>
<td></td>
</tr>
</tbody>
</table>

After Hough (1957).
Figure 7-122. Definition sketch for Coulomb earth force equation.

Figure 7-123. Active earth force for simple Rankine case.
Table 7-16. Coefficients and angles of friction.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Coefficient of Friction, $\mu$</th>
<th>Angle of Wall Friction, $\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stone - Brick - Concrete</td>
<td></td>
<td></td>
</tr>
<tr>
<td>On Dry Clay</td>
<td>0.50</td>
<td>26° 40'</td>
</tr>
<tr>
<td>On Wet or Moist Clay</td>
<td>0.33</td>
<td>18° 20'</td>
</tr>
<tr>
<td>On Sand</td>
<td>0.40</td>
<td>21° 50'</td>
</tr>
<tr>
<td>On Gravel</td>
<td>0.60</td>
<td>31° 00'</td>
</tr>
</tbody>
</table>

NOTE: Angle of friction should be reduced by about 5 degrees if the wall fill will support train or truck traffic; the coefficient $\mu$ would then equal the tangent of the new angle $\delta$.

3. Cohesive Soils.

Sections 1 and 2 above have briefly dealt with forces in cohesionless soil. A cohesive backfill which reduces the active force may be advantageous. However, unless the soil can move continuously to maintain the cohesive resistance, it may relax. Thus, walls should usually be designed for the active force in cohesionless soil.

4. Structures of Irregular Section.

Earth force against structures of irregular section such as stepped-stone blocks or those having two or more back batters may be estimated using equations (7-142) and (7-144) by substituting an approximate average wall batter or slope to determine the angle $\theta$.

5. Submerged Material.

Forces due to submerged fills may be calculated by substituting the unit weight of the material reduced by buoyancy for the value of $w$ in the preceding equations and then adding to the calculated forces the full hydrostatic force due to the water. Values of unit weight for dry, saturated, and submerged materials are indicated in Table 7-15.

6. Uplift Forces.

For design computations, uplift forces should be considered as full hydrostatic force for walls whose bases are below design water level or for walls with saturated backfill.


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CHAPTER 8

Engineering Analysis: Case Study

Redondo-Malaga Cove, California, 23 January 1973
CHAPTER 8
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   a. Design Hurricanes  8-7
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</table>

1. Material Characteristics
   a. Native Sand
   b. Borrow--Source A
   c. Borrow--Source B
CHAPTER 8
ENGINEERING ANALYSIS: CASE STUDY

I. INTRODUCTION

This chapter presents as examples of the techniques presented in this manual a series of calculations for the preliminary design of a hypothetical offshore island in the vicinity of Delaware Bay. The problem serves to illustrate the interrelationships among many types of problems encountered in coastal engineering. The text progresses from development of the physical environment through a preliminary design of several elements of the proposed structure.

For brevity, the design calculations are incomplete; however, when necessary, the nature of additional work required to complete the design is indicated. It should be pointed out that a project of the scope illustrated here would require extensive model testing to verify and supplement the analysis. The design and analysis of such tests is beyond the scope of this manual. In addition, extensive field investigations at the island site would be required to establish the physical environment. These studies would include a determination of engineering and geological characteristics of local sediments, as well as measurement of waves and currents. The results of these studies would then have to be evaluated before beginning a final design.

While actual data for the Delaware Bay site were used when available, specific numbers used in the calculations should not be construed as directly applicable to other design problems in the Delaware Bay area.

II. STATEMENT OF PROBLEM

A 300-acre artificial offshore island is proposed in the Atlantic Ocean just outside the mouth of Delaware Bay. The following are required: (1) characterization of the physical environment at the proposed island site and (2) a preliminary design for the island. Reference is made throughout this chapter to appropriate sections of the Shore Protection Manual.

III. PHYSICAL ENVIRONMENT

1. Site Description.

Figures 8-1 through 8-5 present information on the general physical conditions at the proposed island site. Site plans showing the island location, surrounding shorelines, and bathymetry are given.
Figure 8-1. Location plan, offshore island.
Figure 8-3. Perspective view and section through island.
Figure 8-5. Bottom profiles through island site.
2. Water Levels and Currents--Storm Surge and Astronomical Tides.

The following calculations establish design water levels at the island site using the methods of Chapter 3 and supplemented by data for the Delaware Bay area given in Bretschneider (1959) and U.S. National Weather Service (formerly U.S. Weather Bureau) (1957).


Hurricane A

Radius to maximum winds = \( R = 62.04 \text{ km (33.5 nmi)} \)

Central pressure \( \Delta P = 55.88 \text{ mm Hg (2.2 in. Hg)} \)

Forward speed \( V_F = 27.78 \text{ to } 46.30 \text{ km/hr} \)

(15 to 25 knots)

(u use \( V_F = 46.30 \text{ km/hr} \))

Maximum gradient windspeed (eq. 3-63a)

\[
U_{max} = 0.447 \left[ 14.5 \left( p_n - p_o \right)^{1/2} - R(0.31)f \right]
\]

where for latitude 40 degrees N

\( f = 0.337 \)

\[
U_{max} = 0.447 \left[ 14.5 \left( 55.88 \right)^{1/2} - 62.04(0.31)(0.337) \right]
\]

\( U_{max} = 45.55 \text{ m/s (163.98 km/hr)} \)

Maximum sustained windspeed (eq. 3-62) for \( V_F = 46.3 \text{ km/hr} \)

\[
U_R = 0.865 U_{max} + 0.5 V_F
\]

\[
U_R = 0.865 \left( 163.98 \right) + 0.5 \left( 46.3 \right)
\]

\( U_R = 165 \text{ km/hr} \)

Hurricane B

\( R = 62.04 \text{ km (33.5 nmi)} \)

\( V_F = 46.30 \text{ km/hr (25 knots)} \)

\( U_{max} = 8.05 \text{ km/hr greater than Hurricane A (8.05 km/hr = 2.23 m/s)} \)

Calculate \( \Delta P \) for \( U_{max} = (163.98+8.05) \text{ km/hr} \)

\[
U_{max} = 172.03 \text{ km/hr (47.79 m/s)}
\]

8-7
Figure 8-6. Hurricane storm tracks in the Delaware Bay area.
Rearranging equation (3-63a),

\[
\Delta P = \left\{ \frac{1}{14.5} \left[ \frac{U_{max}}{0.447} + R(0.31f) \right] \right\}^2
\]

\[
\Delta P = \left\{ \frac{1}{14.5} \left[ \frac{47.79}{0.447} + (62.04)(0.31)(0.337) \right] \right\}^2
\]

\[\Delta P = 61.16 \text{ mm Hg}\]

b. Estimate of Storm Surge. Bretschneider (1959) gives an empirical relationship between maximum sustained windspeed and surge height (both pressure- and wind-induced) at the Delaware Bay entrance (applicable only to Delaware Bay). Equation 11 from this reference is used for peak surge (\(S_o\)) computations:

\[S_o = 0.0001 U_R^2 \pm 10\% (U_R \text{ in km/hr})\]

Hurricane A (eq. 3-62)

\[U_R = 0.865 U_{max} + 0.5 V_F\]

\[U_R = 0.865 (163.98) + 0.5 (46.3)\]

\[U_R = 165 \text{ km/hr}\]

\[(S_o)_{max} = 0.0001 (U_R)^2 = 2.72 \text{ m}\]

say \( (S_o)_{max} = 2.75 \pm 0.25 \text{ m}\)

Hurricane B

\[(S_o)_{max} = 0.0001 (172)^2 = 2.96 \text{ m}\]

say \( (S_o)_{max} = 3.00 \pm 0.25 \text{ m}\)

Final results of storm surge estimates from the empirical equation of Bretschneider (1959):

<table>
<thead>
<tr>
<th>Hurricane</th>
<th>((S_o)_{max})</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.75 ± 0.25 m</td>
</tr>
<tr>
<td>B</td>
<td>3.00 ± 0.25 m</td>
</tr>
</tbody>
</table>

c. Observed Water Level Data, Breakwater Harbor, Lewes, Delaware (National Ocean Service (NOS) Tide Tables) (see Ch. 3, Sec. VIII and Table 3-3).

(1) Length of record: 1936 to 1973

(2) Mean tidal range: 1.25 m
(3) Spring range: 1.49 m

(4) Highest observed water levels:
   (a) Average yearly highest: 0.91 m above MHW
   (b) Highest observed: 1.65 m above MHW (6 March 1962)

(5) Lowest observed water levels:
   (a) Average yearly lowest: 0.76 m below MLW
   (b) Lowest observed: 0.91 m below MLW (28 March 1955)

---

**d. Predicted Astronomical Tides.** The probabilities that the water will be above a given level at any time are tabulated for Lewes, Delaware, in Harris (1981), page 164.

The lower limit (LL) of the hour by values are normalized with respect to half the mean range (2.061 ft or 0.628 m). In order to tabulate the elevation above MLW with the corresponding probabilities (see Table 8-1), the following calculation must be done:

\[
2.061 \times (1 + LL) = \text{MLW elevation (ft)}
\]

\[
0.628 \times (1 + LL) = \text{MLW elevation (m)}
\]
Table 8-1. Astronomical tide-water level statistics at Lewes, Delaware.

<table>
<thead>
<tr>
<th>Elevation above MLW, Z (ft)</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.785</td>
<td>0.0000</td>
</tr>
<tr>
<td>5.714</td>
<td>0.0000</td>
</tr>
<tr>
<td>5.643</td>
<td>0.0001</td>
</tr>
<tr>
<td>5.572</td>
<td>0.0005</td>
</tr>
<tr>
<td>5.501</td>
<td>0.0010</td>
</tr>
<tr>
<td>5.431</td>
<td>0.0018</td>
</tr>
<tr>
<td>5.359</td>
<td>0.0028</td>
</tr>
<tr>
<td>5.289</td>
<td>0.0040</td>
</tr>
<tr>
<td>5.218</td>
<td>0.0054</td>
</tr>
<tr>
<td>5.147</td>
<td>0.0072</td>
</tr>
<tr>
<td>5.076</td>
<td>0.0094</td>
</tr>
<tr>
<td>5.005</td>
<td>0.0118</td>
</tr>
<tr>
<td>4.934</td>
<td>0.0147</td>
</tr>
<tr>
<td>4.863</td>
<td>0.0181</td>
</tr>
<tr>
<td>4.792</td>
<td>0.0221</td>
</tr>
<tr>
<td>4.721</td>
<td>0.0269</td>
</tr>
<tr>
<td>4.650</td>
<td>0.0326</td>
</tr>
<tr>
<td>4.579</td>
<td>0.0392</td>
</tr>
<tr>
<td>4.508</td>
<td>0.0464</td>
</tr>
<tr>
<td>4.437</td>
<td>0.0540</td>
</tr>
<tr>
<td>4.366</td>
<td>0.0627</td>
</tr>
<tr>
<td>4.295</td>
<td>0.0717</td>
</tr>
<tr>
<td>4.224</td>
<td>0.0818</td>
</tr>
<tr>
<td>4.153</td>
<td>0.0926</td>
</tr>
<tr>
<td>4.082</td>
<td>0.1038</td>
</tr>
<tr>
<td>4.011</td>
<td>0.1162</td>
</tr>
<tr>
<td>3.969</td>
<td>0.1200</td>
</tr>
<tr>
<td>3.919</td>
<td>0.1556</td>
</tr>
<tr>
<td>3.728</td>
<td>0.1694</td>
</tr>
<tr>
<td>3.656</td>
<td>0.1840</td>
</tr>
<tr>
<td>3.586</td>
<td>0.1991</td>
</tr>
<tr>
<td>3.515</td>
<td>0.2146</td>
</tr>
<tr>
<td>3.444</td>
<td>0.2303</td>
</tr>
<tr>
<td>3.373</td>
<td>0.2462</td>
</tr>
<tr>
<td>3.302</td>
<td>0.2623</td>
</tr>
<tr>
<td>3.231</td>
<td>0.2783</td>
</tr>
<tr>
<td>3.160</td>
<td>0.2947</td>
</tr>
<tr>
<td>3.089</td>
<td>0.3103</td>
</tr>
<tr>
<td>3.018</td>
<td>0.3255</td>
</tr>
<tr>
<td>2.946</td>
<td>0.3407</td>
</tr>
<tr>
<td>2.876</td>
<td>0.3553</td>
</tr>
<tr>
<td>2.805</td>
<td>0.3693</td>
</tr>
<tr>
<td>2.734</td>
<td>0.3826</td>
</tr>
<tr>
<td>2.663</td>
<td>0.3959</td>
</tr>
<tr>
<td>2.592</td>
<td>0.4090</td>
</tr>
<tr>
<td>2.521</td>
<td>0.4215</td>
</tr>
<tr>
<td>2.450</td>
<td>0.4335</td>
</tr>
<tr>
<td>2.379</td>
<td>0.4457</td>
</tr>
<tr>
<td>2.308</td>
<td>0.4576</td>
</tr>
<tr>
<td>2.237</td>
<td>0.4697</td>
</tr>
<tr>
<td>2.166</td>
<td>0.4815</td>
</tr>
<tr>
<td>2.095</td>
<td>0.4935</td>
</tr>
<tr>
<td>2.024</td>
<td>0.5049</td>
</tr>
</tbody>
</table>
e. **Design Water Level Summary.** For purposes of the design problem the following water levels will be used. The criteria used here should not be assumed generally applicable since design water level criteria will vary with the scope and purpose of a particular project.

1. Astronomical tide: use + 1.5 m (MLW) (exceeded 1 percent of time)
2. Storm surge: use + 3.0 m
3. Wave setup: a function of wave conditions

Table 8-2. Tidal currents at Delaware Bay entrance (surface currents only), 1948 values.\(^1\)

<table>
<thead>
<tr>
<th>Time</th>
<th>Velocity(^2) km/hr</th>
<th>Velocity(^2) m/s</th>
<th>Direction (degrees N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flood</td>
<td>-2 hr</td>
<td>1.48</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>-1 hr</td>
<td>2.59</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>Flood</td>
<td>2.96</td>
<td>0.82</td>
</tr>
<tr>
<td>Flood</td>
<td>+1 hr</td>
<td>2.41</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>+2 hr</td>
<td>1.11</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>+3 hr</td>
<td>0.56</td>
<td>0.16</td>
</tr>
<tr>
<td>Ebb</td>
<td>-2 hr</td>
<td>2.41</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>-1 hr</td>
<td>3.89</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>Ebb</td>
<td>4.63</td>
<td>1.29</td>
</tr>
<tr>
<td>Ebb</td>
<td>+1 hr</td>
<td>4.44</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>+2 hr</td>
<td>3.33</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>+3 hr</td>
<td>1.11</td>
<td>0.31</td>
</tr>
</tbody>
</table>

\(^{1}\) From NOS Tidal Current Charts for Delaware Bay and River (1948 and 1960) and NOS Tide Tables.

\(^{2}\) For spring tides.

Example charts from National Ocean Service (NOS) (1948 and 1960) and a summary of tidal current velocities are given in Figures 8-7 through 8-10 are given on the following pages.

3. **Wave Conditions.**

a. **Wave Conditions on Bay Side of Island** (see Ch. 3, Sec. V). Wave data on waves generated in Delaware Bay are not available for the island site. Consequently, wind data and longest fetch shallow-water wave forecasting techniques will be used to estimate wave conditions.

The longest fetch at the Delaware Bay entrance \( F = 89.3 \text{ km} \) (see Figure 8-11).
Figure 8-7. Tidal current chart-maximum flood at Delaware Bay Entrance.
Figure 8-8. Tidal current chart-maximum ebb at Delaware Bay Entrance.
Figure 8-9. Polar diagram of tidal currents at island site.
Figure 8-10. Time variation of tidal current speed at island site.
Figure 8-11. Determination of longest fetch: island site at Delaware Bay entrance.
(1) Significant Wave Height and Period (Wind From NNW Along Central Radial) (see Ch. 3, Sec. VI.1).

Average Depth Along Central Radial

![Graph showing depth and chart data along central radial.]

Significant wave height (eq. 3-39):

\[
H_s = \frac{0.283 \, U_A^2}{g} \tanh \left[ 0.530 \left( \frac{gd}{U_A^2} \right)^{3/4} \right] \tanh \left[ \frac{0.00565}{\tanh \left[ 0.530 \left( \frac{gd}{U_A^2} \right)^{3/4} \right]} \left( \frac{gF}{U_A^2} \right)^{1/2} \right]
\]

Significant wave period (eq. 3-40):

\[
T_s = \frac{7.54 \, U_A}{g} \tanh \left[ 0.833 \left( \frac{gd}{U_A^2} \right)^{3/8} \right] \tanh \left[ \frac{0.0379}{\tanh \left[ 0.833 \left( \frac{gd}{U_A^2} \right)^{3/8} \right]} \left( \frac{gF}{U_A^2} \right)^{1/3} \right]
\]
where

\[ U_A = \text{adjusted wind stress factor} = 0.71 U_s^{1.23} \quad \text{(eq. 3-28a)} \]

\[ U_s = \text{surface windspeed} \]

(2) **Example Calculation.**

\[ U = 80 \text{ km/hr} \quad (22.22 \text{ m/s}) \]
\[ F = 89.3 \text{ km} \quad (89,300 \text{ m}) \]
\[ D = 0.01 \text{ km} \quad (10.37 \text{ m}) \]

\[ U_A = 0.71 U^{1.23} = 0.71 (22.22)^{1.23} = 32.19 \text{ m/s} \]

\[ \frac{gF}{U_A^2} = \frac{(9.806)(89,300)}{(32.19)^2} = 845.09 \]

\[ \frac{gd}{U_A^2} = \frac{(9.806)(10.37)}{(32.19)^2} = 0.0981 \]

\[ H_s = \frac{0.283(32.19)^2}{9.806} \tanh \left[ 0.530 \left(0.0981\right)^{3/4} \right] x \]

\[ \tanh \left\{ \frac{0.00565 (845.09)^{1/2}}{\tanh \left(0.53 \left(0.0981\right)^{3/4} \right)} \right\} \quad \text{(eq. 3-39)} \]

\[ H_s = 2.61 \text{ m} \]

\[ T_s = \frac{7.54 (32.19)}{9.806} \tanh \left[ 0.833 \left(0.0981\right)^{3/8} \right] x \]

\[ \tanh \left\{ \frac{0.0379 (845.09)^{1/3}}{\tanh \left(0.833 \left(0.0981\right)^{3/8} \right)} \right\} \quad \text{(eq. 3-40)} \]

\[ T_s = 6.55 \text{ s} \]

See tabulation and graph on next page.
when $F = 89,300 \text{ m}$ and $d = 10.37 \text{ m}$,

<table>
<thead>
<tr>
<th>$U$ (km/hr)</th>
<th>$U$ (m/s)</th>
<th>$U_A$ (m/s)</th>
<th>$H_S$ (m)</th>
<th>$T_S$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>22.2</td>
<td>32.19</td>
<td>2.61</td>
<td>6.55</td>
</tr>
<tr>
<td>90</td>
<td>25.0</td>
<td>37.22</td>
<td>2.85</td>
<td>6.86</td>
</tr>
<tr>
<td>100</td>
<td>27.7</td>
<td>42.36</td>
<td>3.07</td>
<td>7.14</td>
</tr>
<tr>
<td>110</td>
<td>30.5</td>
<td>47.63</td>
<td>3.29</td>
<td>7.40</td>
</tr>
<tr>
<td>120</td>
<td>33.3</td>
<td>53.02</td>
<td>3.49</td>
<td>7.65</td>
</tr>
<tr>
<td>130</td>
<td>36.1</td>
<td>58.50</td>
<td>3.68</td>
<td>7.89</td>
</tr>
<tr>
<td>140</td>
<td>38.8</td>
<td>64.08</td>
<td>3.86</td>
<td>8.11</td>
</tr>
<tr>
<td>150</td>
<td>41.6</td>
<td>69.76</td>
<td>4.04</td>
<td>8.32</td>
</tr>
<tr>
<td>160</td>
<td>44.4</td>
<td>75.52</td>
<td>4.22</td>
<td>8.52</td>
</tr>
<tr>
<td>170</td>
<td>47.2</td>
<td>81.37</td>
<td>4.38</td>
<td>8.71</td>
</tr>
</tbody>
</table>

( eqs. 3-39 and 3-40 )
(2) **Frequency Analysis.**

(a) **Wind Data.** Wind roses for the Delaware Bay area are given in Figure 8-12 (U.S. Army Engineer District, Philadelphia, 1970). Assume that sizeable waves occur primarily when wind is blowing along central radial from the NW. This is the predominant wind direction for the Delaware Bay area. Wind is from the NW approximately 16 percent of the time.

The maximum observed wind in 18 years of record was 113-km/hr (70-mph) gale from the NW (daily maximum 5-minute windspeed).

(b) **Thom's Fastest-Mile Wind Frequency.** In the absence of tabulated wind data (other than that given on the following page), the windspeed frequencies of Thom (1960), adjusted for wind direction, will be used. Thom's windspeed are multiplied by 0.16 to adjust for direction. This assumes that winds from the NW are distributed the same as are winds when all directions are considered.

<table>
<thead>
<tr>
<th>Quantile</th>
<th>Recurrence Interval (years)</th>
<th>Adjusted(^1) Recurrence Interval (years)</th>
<th>(U^2) mph</th>
<th>(U^3) km/hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>2</td>
<td>12.5</td>
<td>55</td>
<td>88.5</td>
</tr>
<tr>
<td>0.02</td>
<td>50</td>
<td>312.5</td>
<td>90</td>
<td>144.8</td>
</tr>
<tr>
<td>0.01</td>
<td>100</td>
<td>625.0</td>
<td>100</td>
<td>160.9</td>
</tr>
</tbody>
</table>

1 Adjusted for direction (column 2 divided by 0.16).
2 Extreme fastest-mile windspeed.
3 Extreme fastest-km wind = 1.6093 x \(U\) fastest-mile windspeed.
Figure 8-12. Wind data in the vicinity of Delaware Bay.
(c) Duration (t) of Fastest-Mile Wind.

\[
t = \frac{1 \text{ km}}{U \text{ (km/hr)}} \times \frac{60 \text{ min}}{\text{hr}} \left( \frac{1 \text{ mile}}{U \text{ (mph)}} \times \frac{60 \text{ min}}{\text{hr}} \right)
\]

\[t = \text{duration of wind in minutes}\]

Since the durations under consideration here are not sufficiently long to generate maximum wave conditions, Thorn's wind data will result in a high estimate of wave heights and periods. The dashed line on Figure 8-13 will be used to establish frequency of occurrence of given wave conditions; calculated wave height recurrence intervals will be conservative.
Figure 8-13. Probability distribution of maximum windspeed: Thom’s fastest-mile wind.
From the dashed curve in Figure 8-13 and graph on page 8-20, for $H_s$ and $T_s$ as a function of $U$ find the following:

The computed wave heights plot as a straight line on log-normal probability paper (see Fig 8-14).

Economic considerations as well as the purpose of a given structure will determine the design wave conditions. The increased protection afforded by designing for a higher wave would have to be weighed against the increase in structure cost.

For the illustrative purposes of this problem, the significant wave height with a recurrence interval of 100 years will be used. Therefore, for design,

$$H_s = 1.09 \text{ m (3.59 ft)}$$

$$T_s = 7.78 \text{ s}$$

for waves generated in Delaware Bay.

<table>
<thead>
<tr>
<th>Recurrence Interval (years)</th>
<th>Probability of Exceedance</th>
<th>$U$ (km/hr)</th>
<th>$H_s$ (m)</th>
<th>$T_s$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.5</td>
<td>64.4</td>
<td>2.21</td>
<td>6.00</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
<td>77.2</td>
<td>2.55</td>
<td>6.46</td>
</tr>
<tr>
<td>10</td>
<td>0.1</td>
<td>86.9</td>
<td>2.78</td>
<td>6.76</td>
</tr>
<tr>
<td>20</td>
<td>0.05</td>
<td>96.6</td>
<td>3.00</td>
<td>7.05</td>
</tr>
<tr>
<td>50</td>
<td>0.02</td>
<td>111.0</td>
<td>3.31</td>
<td>7.43</td>
</tr>
<tr>
<td>100</td>
<td>0.01</td>
<td>125.5</td>
<td>3.59</td>
<td>7.78</td>
</tr>
<tr>
<td>200</td>
<td>0.005</td>
<td>138.4</td>
<td>3.83</td>
<td>8.07</td>
</tr>
</tbody>
</table>

Figure 8-14. Frequency of occurrence of significant wave heights for waves generated in Delaware Bay.
b. Wave Conditions on Ocean Side of Island: Hindcast wave statistics are available for several U.S. east coast locations in Corson, et al. (1981), Corson et al. (1982), and Jensen (1983). Data are available from the mouth of Delaware Bay; but deepwater wave data are chosen for statistical analysis to demonstrate the method of transforming data from deep water to another location in shallow water (i.e., the island site). (See Figure 8-15 for station 4 location and Table 8-4 for data.)

(1) Idealized Refraction Analysis (see Ch. 2, Sec. III). For purposes of this problem, refraction by straight parallel bottom contours will be assumed.

Azimuth of shoreline = 30° (see Fig. 8-17)

(2) Wave Directions.

<table>
<thead>
<tr>
<th>Direction of Wave Approach</th>
<th>Angle Between Wave Direction and Shoreline (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NNE</td>
<td>-7.5 (α_o &gt; 90, neglect)</td>
</tr>
<tr>
<td>NE</td>
<td>+15.0 α_o = 75°</td>
</tr>
<tr>
<td>ENE</td>
<td>+37.5 α_o = 52.5</td>
</tr>
<tr>
<td>E</td>
<td>+60.0 α_o = 30.0</td>
</tr>
<tr>
<td>ESE</td>
<td>+82.5 α_o = 7.5</td>
</tr>
<tr>
<td>SE</td>
<td>+105.0 α_o = 15.0</td>
</tr>
<tr>
<td>SSE</td>
<td>+127.5 α_o = 37.5</td>
</tr>
<tr>
<td>S3</td>
<td>+150.0 α_o = 60.0</td>
</tr>
<tr>
<td>SSW</td>
<td>+172.5 α_o = 82.5</td>
</tr>
<tr>
<td>SW</td>
<td>+195.0 α_o &gt; 90, neglect)</td>
</tr>
</tbody>
</table>

The hindcast statistics are available for the Atlantic coast and the Great Lakes. They will be available for the Pacific and gulf coasts at a future date.

α_o is the angle between the direction of wave approach and a normal to the shoreline.

Used for typical refraction calculations given on following pages.
Figure 8-15. Station 4 location.

STATION 4
37.3°N 72.6°W
241 KM TO SHORE
DEPTH = 2800 METERS
MEAN $H_s$ = 1.8 METERS
MAXIMUM $H_s$ = 13.1 METERS

(CORSON ET AL., 1981)
Table 8-4. Hindcast wave statistics for station 4.1

<table>
<thead>
<tr>
<th>Direction (deg)</th>
<th>Duration (hr) for These Periods 2</th>
<th>Total Duration (hr)</th>
<th>Wave Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3-4.9s 5-6.9s 7-8.9s 9-10.9s 11-12.9s 13-14.9s 15-16.9s 17-18.9s 19-20.9s 21-22.9s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
<td>25 24 12</td>
<td>91</td>
<td>0 - 0.49</td>
</tr>
<tr>
<td>60-89.9</td>
<td>23 22 2</td>
<td>69</td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td>28 20 2</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td>19 40 16 2</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td>30 57 27 2</td>
<td>116</td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td>44 23 6 3</td>
<td>76</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOTAL FOR 0- TO 0.49-m WAVE HEIGHT:</td>
<td>439</td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
<td>71 10 126 107 5</td>
<td>319</td>
<td>0.50-0.99</td>
</tr>
<tr>
<td>60-89.9</td>
<td>54 11 75 19</td>
<td>159</td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td>50 5 59 13 1</td>
<td>128</td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td>45 7 95 23 3</td>
<td>173</td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td>79 12 84</td>
<td>175</td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td>TOTAL FOR 0.50- TO 0.99-m WAVE HEIGHT:</td>
<td>956</td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
<td>6 49 41 57 10</td>
<td>163</td>
<td>1.00-1.49</td>
</tr>
<tr>
<td>60-89.9</td>
<td>9 38 45 11</td>
<td>103</td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td>11 26 47 11</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td>10 25 69 29 9</td>
<td>142</td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td>8 45 76 53 26 2</td>
<td>210</td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td>13 108 93 44 10 1</td>
<td>269</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOTAL FOR 1.00- TO 1.49-m WAVE HEIGHT:</td>
<td>982</td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
<td>1 31 16 16 4</td>
<td>68</td>
<td>1.50-1.99</td>
</tr>
<tr>
<td>60-89.9</td>
<td>20 18 10 2</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td>15 27 9 1</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td>18 44 32 13 1</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td>32 56 42 23 4</td>
<td>157</td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td>1 60 82 57 22 3 2</td>
<td>227</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOTAL FOR 1.50- TO 1.99-m WAVE HEIGHT:</td>
<td>962</td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
<td>9 15 6</td>
<td>30</td>
<td>2.00-2.49</td>
</tr>
<tr>
<td>60-89.9</td>
<td>10 21 7</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td>6 20 6 1 1</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td>7 33 26 11 3</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td>10 38 24 15 6 1</td>
<td>94</td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td>17 55 46 27 5 1</td>
<td>165</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOTAL FOR 2.00- TO 2.49-m WAVE HEIGHT:</td>
<td>421</td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
<td>1 24 1 1</td>
<td>27</td>
<td>2.50-2.99</td>
</tr>
<tr>
<td>60-89.9</td>
<td>1 29 4 2</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td>2 26 6 2</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td>2 33 16 12 2</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td>1 37 19 14 6 1</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td>1 60 20 19 3 1</td>
<td>104</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOTAL FOR 2.50- TO 2.99-m WAVE HEIGHT:</td>
<td>344</td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
<td>24 2 1</td>
<td>27</td>
<td>3.00-3.49</td>
</tr>
<tr>
<td>60-89.9</td>
<td>20 2 1</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td>13 5 1</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td>17 10 6 1</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td>22 5 10 4 1</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td>39 10 15 5 1</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOTAL FOR 3.00- TO 3.49-m WAVE HEIGHT:</td>
<td>215</td>
<td></td>
</tr>
</tbody>
</table>

1 from Corson et al. (1981).
2 Only durations > 1hr are shown.
Table 8-4. Hindcast wave statistics for station 4 (continued).

<table>
<thead>
<tr>
<th>Direction (deg)</th>
<th>Duration (hr) for These Periods</th>
<th>Total Duration (hr)</th>
<th>Wave Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3-4.9s 5-6.9s 7-8.9s 9-10.9s 11-12.9s 13-14.9s 15-16.9s 17-18.9s 19-20.9s 21-22.9s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
<td>17  1  1</td>
<td>18</td>
<td>3.50-3.99</td>
</tr>
<tr>
<td>60-89.9</td>
<td>10  3  1</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td>4   3  1</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td>5   5  4</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td>9   3  4</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td>25  7  6</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOTAL FOR 3.50- TO 3.99-m WAVE HEIGHT:</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
<td>11  4</td>
<td>15</td>
<td>4.00-4.49</td>
</tr>
<tr>
<td>60-89.9</td>
<td>6   3</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td>2   2</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td>2   2</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td>3   3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td>13  10 3</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOTAL FOR 4.00- TO 4.49-m WAVE HEIGHT:</td>
<td>76</td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
<td>3   8</td>
<td>11</td>
<td>4.50-4.99</td>
</tr>
<tr>
<td>60-89.9</td>
<td>3   6</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td>1   1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td>1   1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td>1   2</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td>3   7</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOTAL FOR 4.50- TO 4.99-m WAVE HEIGHT:</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
<td>2   7</td>
<td>9</td>
<td>5.00-5.49</td>
</tr>
<tr>
<td>60-89.9</td>
<td>2   3</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td>1   1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td>1   1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td>1   1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td>2   4</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOTAL FOR 5.00- TO 5.49-m WAVE HEIGHT:</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
<td>1   6</td>
<td>7</td>
<td>5.50-5.99</td>
</tr>
<tr>
<td>60-89.9</td>
<td>1   3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td>1   1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td>1   1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td>1   3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td>1   3</td>
<td>5</td>
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</tr>
<tr>
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<td>19</td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
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</tr>
<tr>
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<td>1</td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td>1   1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td>1   1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td>1   1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td>1   1</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOTAL FOR 6.00- TO 6.49-m WAVE HEIGHT:</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
<td>1   1</td>
<td>3</td>
<td>6.50-6.99</td>
</tr>
<tr>
<td>60-89.9</td>
<td>1   1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td>1   1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td>1   1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td>1   1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td>1   1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TOTAL FOR 6.50- TO 6.99-m WAVE HEIGHT:</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

2 Only durations > 1hr are shown.
Table 8-4. Hindcast wave statistics for station 4 (concluded).

<table>
<thead>
<tr>
<th>Direction (deg)</th>
<th>Duration (hr) for These Periods</th>
<th>Total Duration (hr)</th>
<th>Wave Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3-4.9s 5-6.9s 7-8.9s 9-10.9s 11-12.9s 13-14.9s 15-16.9s 17-18.9s 19-20.9s 21-22.9s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-59.9</td>
<td>2 2 1 1</td>
<td>4</td>
<td>7.00-7.49</td>
</tr>
<tr>
<td>60-89.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL FOR 7.00- TO 7.49-m WAVE HEIGHT:</td>
<td></td>
<td>2</td>
<td>7.50-7.99</td>
</tr>
<tr>
<td>30-59.9</td>
<td>1 1</td>
<td>2</td>
<td>7.50-7.99</td>
</tr>
<tr>
<td>60-89.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL FOR 7.50- TO 7.99-m WAVE HEIGHT:</td>
<td></td>
<td>1</td>
<td>8.00-8.49</td>
</tr>
<tr>
<td>30-59.9</td>
<td>1</td>
<td>1</td>
<td>8.00-8.49</td>
</tr>
<tr>
<td>60-89.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL FOR 8.00- TO 8.49-m WAVE HEIGHT:</td>
<td></td>
<td>1</td>
<td>8.50-8.99</td>
</tr>
<tr>
<td>30-59.9</td>
<td>1</td>
<td>1</td>
<td>8.50-8.99</td>
</tr>
<tr>
<td>60-89.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL FOR 8.50- TO 8.99-m WAVE HEIGHT:</td>
<td></td>
<td>1</td>
<td>9.00-9.49</td>
</tr>
<tr>
<td>30-59.9</td>
<td>1</td>
<td>1</td>
<td>9.00-9.49</td>
</tr>
<tr>
<td>60-89.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL FOR 9.00- TO 9.49-m WAVE HEIGHT:</td>
<td></td>
<td>1</td>
<td>9.50-9.99</td>
</tr>
<tr>
<td>30-59.9</td>
<td>1</td>
<td>1</td>
<td>9.50-9.99</td>
</tr>
<tr>
<td>60-89.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90-119.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120-149.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150-179.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>180-209.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL FOR 9.50 TO 9.99-m WAVE HEIGHT:</td>
<td></td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

2 Only durations > 1hr are shown.
Figure 8-16. Wave diagram for station 4 off the Delaware Bay entrance (numbers within concentric rings are percent of waves of different height occurring from each direction; numbers in triangles represent percent of time waves occur in each direction).
Figure 8-17. General shoreline alignment in vicinity of Delaware Bay for refraction analysis.
(3) Typical Refraction Calculations. Use $d = 12.0$ m at structure.

Shoaling Coefficient:

$$K_g = \frac{H}{H'} = \left[ \frac{\coth \left( \frac{2\pi d}{L} \right)}{1 + \frac{4\pi d/L}{\sinh \left( \frac{4\pi d/L}{L} \right)}} \right]^{1/2} \quad (eq. 2-44)$$

equivalently,

$$K_g = \left( \frac{C_o}{2\pi C} \right)^{1/2} = \left( \frac{gT^2}{4\pi nL} \right)^{1/2}$$

where

$H$ = wave height

$H'$ = deepwater wave height equivalent to observed shallow-water wave if unaffected by refraction and friction

$L$ = wavelength

$C$ = wave velocity

$C_o$ = deepwater wave velocity

$T$ = wave period

Refraction coefficient and angle:

$$\sin \alpha = \left( \frac{C}{C_o} \right) \sin \alpha_o \quad (eq. 2-78b)$$

Note that equation (2-78b) is written between deep water and $d = 12.0$ m, since bottom contours and shoreline have been assumed straight and parallel. For straight parallel bottom contours, the expression for the refraction coefficient reduces to

$$K_R = \left( \frac{b_o}{b} \right)^{1/2} = \left( \frac{\cos \alpha}{\cos \alpha_o} \right)^{1/2}$$

where

$b$ = spacing between wave orthogonals

$b_o$ = deepwater orthogonal spacing
Recall,

\[ L_o = \frac{gT^2}{2\pi} \quad \text{deepwater wavelength in meters} \quad \text{(eq. 2-8a)} \]

and

\[ \frac{L}{L_o} = \frac{C}{C_o} = \tanh \left( \frac{2\pi d}{L} \right) \quad \text{(eq. 2-11)} \]

Typical refraction-shoaling calculations are given in the tabulation below. Calculations for various directions and for a range of periods follow (see Tables 8-5 and 8-6).

The following tabulates the results of example calculations for waves between 150 and 179.9 degrees from North (angle between direction of wave approach and normal to the shoreline in deep water = \( \alpha_o = 45 \) degrees) ; \( d = 12.0 \) m .

<table>
<thead>
<tr>
<th>( T ) (s)</th>
<th>( L_o ) (m)</th>
<th>( d/L_o )</th>
<th>( H/H_o )</th>
<th>( C/L_o )</th>
<th>( \alpha ) (deg)</th>
<th>( K_r )</th>
<th>( K_sK_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>25.0</td>
<td>0.4806</td>
<td>0.98856</td>
<td>0.99536</td>
<td>44.7</td>
<td>0.9977</td>
<td>0.9863</td>
</tr>
<tr>
<td>6</td>
<td>56.2</td>
<td>0.2136</td>
<td>0.92142</td>
<td>0.90270</td>
<td>39.7</td>
<td>0.9584</td>
<td>0.8831</td>
</tr>
<tr>
<td>8</td>
<td>99.9</td>
<td>0.1201</td>
<td>0.92036</td>
<td>0.75913</td>
<td>32.5</td>
<td>0.9155</td>
<td>0.8426</td>
</tr>
<tr>
<td>10</td>
<td>156.1</td>
<td>0.0769</td>
<td>0.95926</td>
<td>0.63887</td>
<td>26.9</td>
<td>0.8903</td>
<td>0.8540</td>
</tr>
<tr>
<td>12</td>
<td>244.7</td>
<td>0.0534</td>
<td>1.01180</td>
<td>0.54670</td>
<td>22.7</td>
<td>0.8756</td>
<td>0.8860</td>
</tr>
<tr>
<td>14</td>
<td>305.9</td>
<td>0.0392</td>
<td>1.06800</td>
<td>0.47580</td>
<td>19.7</td>
<td>0.8670</td>
<td>0.9255</td>
</tr>
<tr>
<td>16</td>
<td>399.5</td>
<td>0.0300</td>
<td>1.12500</td>
<td>0.42050</td>
<td>17.3</td>
<td>0.8610</td>
<td>0.9682</td>
</tr>
<tr>
<td>18</td>
<td>505.7</td>
<td>0.0237</td>
<td>1.18130</td>
<td>0.37632</td>
<td>15.4</td>
<td>0.8565</td>
<td>1.0118</td>
</tr>
</tbody>
</table>

Column 2

<table>
<thead>
<tr>
<th>Source of Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>From equation (2-8a).</td>
</tr>
<tr>
<td>12.0 m divided by column (2).</td>
</tr>
<tr>
<td>Equation (2-44) or Table C-1, Appendix C.</td>
</tr>
<tr>
<td>Table C-1, Appendix C: ( C/C_o = \tanh (2\pi d/L) ).</td>
</tr>
<tr>
<td>Equation (2-78b)</td>
</tr>
<tr>
<td>( K_r = \left( \frac{\cos \alpha_o}{\cos \alpha} \right)^{1/2} ).</td>
</tr>
<tr>
<td>Column (4) times column (7).</td>
</tr>
</tbody>
</table>

\( K_sK_r \) can also be obtained from Plate C-6, Appendix C.
Table 8-5. Breaker angles and refraction and shoaling coefficients in $d = 12 \text{ m}$.

<table>
<thead>
<tr>
<th>$\alpha_o$</th>
<th>$\alpha$ (deg)</th>
<th>$\epsilon$</th>
<th>$\eta$</th>
<th>$\zeta$</th>
<th>$\zeta'\eta'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 deg</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>$T$ (s)</td>
<td>14.92</td>
<td>13.51</td>
<td>11.33</td>
<td>9.52</td>
<td>8.13</td>
</tr>
<tr>
<td>$K_\varepsilon$</td>
<td>0.9886</td>
<td>0.9214</td>
<td>0.9204</td>
<td>0.9593</td>
<td>1.012</td>
</tr>
<tr>
<td>$K_\eta'$</td>
<td>0.9998</td>
<td>0.9967</td>
<td>0.9925</td>
<td>0.9897</td>
<td>0.9878</td>
</tr>
<tr>
<td>$K_\varepsilon K_\eta'$</td>
<td>0.9884</td>
<td>0.9184</td>
<td>0.9135</td>
<td>0.9493</td>
<td>0.9995</td>
</tr>
<tr>
<td>45 deg</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>$T$ (s)</td>
<td>44.73</td>
<td>39.67</td>
<td>32.47</td>
<td>26.86</td>
<td>22.74</td>
</tr>
<tr>
<td>$K_\varepsilon$</td>
<td>0.9886</td>
<td>0.9214</td>
<td>0.9204</td>
<td>0.9593</td>
<td>1.012</td>
</tr>
<tr>
<td>$K_\eta'$</td>
<td>0.9977</td>
<td>0.9584</td>
<td>0.9155</td>
<td>0.8903</td>
<td>0.8756</td>
</tr>
<tr>
<td>$K_\varepsilon K_\eta'$</td>
<td>0.9863</td>
<td>0.8831</td>
<td>0.8426</td>
<td>0.8540</td>
<td>0.8860</td>
</tr>
<tr>
<td>75 deg</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>$T$ (s)</td>
<td>74.03</td>
<td>60.67</td>
<td>47.16</td>
<td>38.11</td>
<td>31.88</td>
</tr>
<tr>
<td>$K_\varepsilon$</td>
<td>0.9886</td>
<td>0.9214</td>
<td>0.9204</td>
<td>0.9593</td>
<td>1.012</td>
</tr>
<tr>
<td>$K_\eta'$</td>
<td>0.9710</td>
<td>0.7271</td>
<td>0.6170</td>
<td>0.5735</td>
<td>0.5521</td>
</tr>
<tr>
<td>$K_\varepsilon K_\eta'$</td>
<td>0.9590</td>
<td>0.6699</td>
<td>0.5678</td>
<td>0.5501</td>
<td>0.5586</td>
</tr>
</tbody>
</table>

8-35
Table 8-6. Summary of refraction analyses in \( d = 12 \) m (numbers given in table are \( K_S \) \( K_R \)).

<table>
<thead>
<tr>
<th>Direction from N (deg)</th>
<th>Range (deg)</th>
<th>Angle between wave orthogonal and normal to the shoreline</th>
<th>Wave Period (( \sigma ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>30–59.9</td>
<td>75°</td>
<td>4</td>
</tr>
<tr>
<td>75</td>
<td>60–89.9</td>
<td>45°</td>
<td>6</td>
</tr>
<tr>
<td>105</td>
<td>90–119.9</td>
<td>15°</td>
<td>8</td>
</tr>
<tr>
<td>135</td>
<td>120–149.9</td>
<td>15°</td>
<td>10</td>
</tr>
<tr>
<td>165</td>
<td>150–179.9</td>
<td>45°</td>
<td>12</td>
</tr>
<tr>
<td>195</td>
<td>180–209.9</td>
<td>75°</td>
<td>14</td>
</tr>
</tbody>
</table>

Angle between wave orthogonal and normal to the shoreline.

Refraction-shoaling coefficients are summarized graphically in Figure 8-18 on the next page.

(4) Transformation of Wave Statistics by Refraction and Shoaling. The refraction-shoaling coefficients calculated previously will be used to transform the deepwater wave statistics given in Table 8-4 (see Tables 8-7 and 8-8 and Figure 8-19). The resulting statistics will be only approximations since only the significant wave is considered in the analysis. The actual sea surface is made up of many wave periods or frequencies, each of which results in a different refraction-shoaling coefficient.
Figure 8-18. Refraction-shoaling coefficient as a function of wave direction and wave period.
Table 8-7. Transformed wave heights: significant heights and periods in $d = 12.0$ m.

<table>
<thead>
<tr>
<th>Deepwater Height (m)</th>
<th>$0_0$ (deg)</th>
<th>Angle from North (deg)</th>
<th>Range (deg)</th>
<th>Wave Period (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>75</td>
<td>45</td>
<td>30-59.9</td>
<td>180-209.9</td>
</tr>
<tr>
<td>&lt;9.5</td>
<td>45</td>
<td>75</td>
<td>60-89.9</td>
<td>150-179.9</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>45</td>
<td>30-59.9</td>
<td>180-209.9</td>
</tr>
<tr>
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1 Numbers represent transformed wave height. For example, a 10-meter-high deepwater wave with a period of 14 seconds approaching from N 75 deg E (in deep water) will be 9.255 meters high at the island site (i.e., in a depth of 12.0 meters).

2 Numbers in parentheses represent the number of hours waves are below given height and above next lower height for given period and direction. For example, deepwater waves between 9.5 and 10 meters in height with a period of 12 seconds were experienced for 1 hour in the one year of hindcast data. Equivalently, the wave height at the structure site for the given deepwater wave statistics will be between 5.307 and 5.586 meters for 1 hour.
Table 8-7. Transformed wave heights: significant heights and periods in d = 12.0 m (continued).

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<th>Range (deg)</th>
<th>Wave Period (s)</th>
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</table>

1 Numbers represent transformed wave height. For example, a 10-meter-high deepwater wave with a period of 14 seconds approaching from N 75 deg E (in deep water) will be 9.255 meters high at the island site (i.e., in a depth of 12.0 meters).

2 Numbers in parentheses represent the number of hours waves are below given height and above next lower height for given period and direction. For example, deepwater waves between 9.5 and 10 meters in height with a period of 12 seconds were experienced for 1 hour in the one year of hindcast data. Equivalently, the wave height at the structure site for the given deepwater wave statistics will be between 5.307 and 5.586 meters for 1 hour.
Table 8-7. Transformed wave heights: significant heights and periods in d = 12.0 m (continued).

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<td>(16)</td>
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<td>(12)</td>
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</table>

1 Numbers represent transformed wave height. For example, a 10-meter-high deepwater wave with a period of 14 seconds approaching from N 75 deg E (in deep water) will be 9.255 meters high at the island site (i.e., in a depth of 12.0 meters).

2 Numbers in parentheses represent the number of hours waves are below given height and above next lower height for given period and direction. For example, deepwater waves between 9.5 and 10 meters in height with a period of 12 seconds were experienced for 1 hour in the one year of hindcast data. Equivalently, the wave height at the structure site for the given deepwater wave statistics will be between 5.307 and 5.586 meters for 1 hour.

8-40
Table 8-7. Transformed wave heights: significant heights and periods in d = 12.0 m (concluded).

<table>
<thead>
<tr>
<th>Deepwater Height (m)</th>
<th>Angle from North (deg)</th>
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</table>

1 Numbers represent transformed wave height. For example, a 10-meter-high deepwater wave with a period of 14 seconds approaching from N 75 deg E (in deep water) will be 9.255 meters high at the island site (i.e., in a depth of 12.0 meters).

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The following tabulations are to be used with Table 8-7. The first lists the number of hours waves of a particular height were present at the structure site. (For example, for waves 7 meters high, with a 12-second period from 75 degrees north (from Table 8-7), wave height at the structure was between 7.088 and 6.645 meters for 1 hour. Therefore, wave height was above 7 meters for $1 \times \frac{0.088}{(7.088 - 6.645)} = 0.199$ hour. Wave height between 6 and 7 meters was $1 - 0.199 = 0.801$ hour.) The second tabulation sums hours for a given wave height and associated frequency. Note that the total hours of waves less than 3 meters high is given, although the listing for these waves is either incomplete or not given; these totals were obtained by completing the calculations using the data in Table 8-7.

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</table>

8-42
Total hours in record = 8766

<table>
<thead>
<tr>
<th>Height (m)</th>
<th>Total Hours</th>
<th>Frequency</th>
</tr>
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<tbody>
<tr>
<td>H ≥ 7</td>
<td>1.199</td>
<td>0.000137</td>
</tr>
<tr>
<td>H ≥ 6</td>
<td>5.553</td>
<td>0.000634</td>
</tr>
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<td>H ≥ 5</td>
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<td>0.001836</td>
</tr>
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<td>H ≥ 4</td>
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<td>H ≥ 3</td>
<td>208.307</td>
<td>0.023763</td>
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<tr>
<td>H ≥ 2</td>
<td>769.689</td>
<td>0.087804</td>
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<td>H ≥ 1</td>
<td>2278.767</td>
<td>0.259955</td>
</tr>
<tr>
<td>H ≥ 0</td>
<td>8766</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

1 Number of hours wave height equalled or exceeded given value.
2 1.99 hours/8766 hours = 0.000137.
Table 8-8. Deepwater wave statistics (without consideration of direction).

<table>
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<tr>
<th>Significant Wave Height (m)</th>
<th>Cumulative Hours</th>
<th>Probability of Exceedance</th>
</tr>
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<tr>
<td>&lt; 10.0</td>
<td>1</td>
<td>0.00011</td>
</tr>
<tr>
<td>&lt; 9.5</td>
<td>2</td>
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<td>&lt; 9.0</td>
<td>3</td>
<td>0.00034</td>
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<tr>
<td>&lt; 8.5</td>
<td>4</td>
<td>0.00046</td>
</tr>
<tr>
<td>&lt; 8.0</td>
<td>8</td>
<td>0.00091</td>
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<td>&lt; 7.5</td>
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<td>46</td>
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<td>74</td>
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<td>119</td>
<td>0.01358</td>
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<td>195</td>
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<tr>
<td>&lt; 4.0</td>
<td>315</td>
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<td>530</td>
<td>0.06046</td>
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<tr>
<td>&lt; 3.0</td>
<td>874</td>
<td>0.09970</td>
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<td>0.14773</td>
</tr>
<tr>
<td>&lt; 2.0</td>
<td>1957</td>
<td>0.22325</td>
</tr>
<tr>
<td>&lt; 1.5</td>
<td>2939</td>
<td>0.33527</td>
</tr>
<tr>
<td>&lt; 1.0</td>
<td>3893</td>
<td>0.44410</td>
</tr>
<tr>
<td>&lt; 0.5</td>
<td>8766</td>
<td>1.00000</td>
</tr>
</tbody>
</table>

1 Wave statistics are derived from data given in Corson et al. (1981).

Curves showing deepwater wave height statistics and transformed statistics are given in Figure 8-19.
Figure 8-19. Frequency of occurrence of significant wave heights for waves generated in ocean transformation by refraction and shoaling.
IV. PRELIMINARY DESIGN

1. Selection of Design Waves and Water Levels.

The selection of design conditions is related to the economics of construction and annual maintenance costs to repair structure in the event of extreme wave action. These costs are related to the probability of occurrence of extreme waves and high water levels. There will usually be some design wave height which will minimize the average annual cost (including amortization of first cost). This optimum design wave height will give the most economical design.

Intangible considerations such as the environmental consequences of structural failure or the possibility of loss of life in the event of failure must also enter into the decision of selecting design conditions. These factors are related to the specific purpose of each structure.

The following design conditions are assumed for the illustrative purposes of this problem.

a. Water Levels (MLW datum).

   (1) Storm surge (less astronomical tide): use 3.0 m.

   (2) Astronomical tide (use water level exceeded 1 percent of time): 1.5 m.

   (3) Wave setup (assumed negligible since structure is in relatively deep water and not at beach).
b. **Wave Conditions on Bay Side of Island.**

(1) Use conditions with 100-year recurrence interval:

\[ H_s = 3.59 \text{ m} \]
\[ T_s = 7.78 \text{ s} \]

c. **Wave Conditions on Ocean Side of Island.** From hindcast statistics (wave height exceeded 0.1 percent of the time in shallow water), use

\[ H_s = 6.0 \text{ m} \]

Note that the reciprocal of an exceedance probability associated with a particular wave according to the present hindcast statistics is not the return period of this wave. For structural design purposes, a statistical analysis of extreme wave events is recommended.

2. **Revetment Design: Ocean Side of Island.**

The ocean side of the island will be protected by a revetment using concrete armor units.

a. **Type of Wave Action.** The depth at the site required to initiate breaking to the 6.0-meter design wave is as follows for a slope in front of the structure where \( m = 0 \) (see Ch. 7, Sec. 1):

\[ H_b = 0.78 d_b \]

or

\[ d_b = \frac{H_b}{0.78} = \frac{6.0}{0.78} = 7.7 \text{ m} \]

where \( H_b \) is the breaker height and \( d_b \) is the water depth at the breaking wave.

Since the depth at the structure \( (d_s = 12.0 \text{ m}) \) is greater than the computed breaking depth (7.7 m), the structure will be subjected to non-breaking waves.

b. **Selection Between Alternative Designs.** The choice of one cross section and/or armor unit type over another is primarily an economic design requiring evaluation of the costs of various alternatives. A comparison of several alternatives follows:

Type of Armor Unit: Tribars vs Tetrapods

Structure Slope: 1:1.5, 1:2, 1:2.5, and 1:3

Concrete Unit Weight: 23.56 kN/m³, 25.13 kN/m³, 26.70 kN/m³

The use of concrete armor units will depend on the availability of suitable quarrrystone and on the economics of using concrete as opposed to stone.
(1) Preliminary Cross Section (modified from Figure 7-116).

\[ W_A = \text{WEIGHT OF INDIVIDUAL ARMOR UNIT} \]
\[ W_R = \text{WEIGHT OF PRIMARY COVER LAYER IF MADE OF ROCK} \]
\[ r_A = \text{COVER LAYER THICKNESS} \]
\[ r_1 = \text{THICKNESS OF FIRST UNDERLAYER} \]
\[ \theta = \text{ANGLE OF STRUCTURE FACE RELATIVE TO HORIZONTAL} \]

(2) Crest Elevation. Established by maximum runup. Runup (R) estimate:

\[ H_S = 6 \text{ m} \]
\[ d = 16.5 \text{ m} \]
\[ T = ? \text{ (use point on runup curve giving maximum runup)} \]

\[ \frac{d}{H_S} = \frac{16.5}{6} = 2.75 \text{ (use Fig. 7-20)} \]

<table>
<thead>
<tr>
<th>( \cot \theta )</th>
<th>( \left( \frac{R}{H_S} \right)_{\text{max}} )</th>
<th>R (m)</th>
<th>Crest Elevation (^1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.05</td>
<td>6.3</td>
<td>use 10.8</td>
</tr>
<tr>
<td>2.0</td>
<td>1.10</td>
<td>6.6</td>
<td>use 11.1</td>
</tr>
<tr>
<td>2.5</td>
<td>1.05</td>
<td>6.3</td>
<td>use 10.8</td>
</tr>
<tr>
<td>3.0</td>
<td>1.00</td>
<td>6.0</td>
<td>use 10.5</td>
</tr>
</tbody>
</table>

\(^1\)Waves over 6 m will result in some overtopping.

8-48
(3) Armor Unit Size.

(a) Primary Cover Layer (see Ch. 7, Sec. III.7,a).

\[ W = \frac{w_r H^3}{gK_D \left( S_r - 1 \right)^3 \cot \theta} \]  
(eq. 7-116)

where

\( W \) = mass of armor unit
\( H \) = design wave height = 6 m
\( w_r \) = unit weight of concrete

\( w_r = 23.56 \text{ kN/m}^3, \ 25.13 \text{ kN/m}^3, \text{ and } 26.70 \text{ kN/m}^3 \)

\( \cot \theta \) = structure slope 1.5, 2.0, 2.5, and 3.0

\( S_r = w_r / w_w \) = ratio of concrete unit weight to unit weight of water

\( K_D \) = stability coefficient (depends on type of unit, type of wave action, and structure slope)

The calculations that follow (Tables 8-9 and 8-10 and Figs. 8-20 through 8-25) are for the structure trunk subjected to nonbreaking wave action. Stability coefficients are obtained from Table 7-8.
Table 8-9. Required armor unit weights: structure trunk.

<table>
<thead>
<tr>
<th>Type of Armor Unit</th>
<th>$w'_{m}$ (kN/m$^3$)</th>
<th>Slope (cot $\theta$)</th>
<th>Armor Unit Stability Coefficient, $K_D$</th>
<th>$W_A^{1}$ (metric tons)</th>
<th>Percent$^2$ Damage for 1% Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tribar</td>
<td>23.56</td>
<td>1.5</td>
<td>10.0</td>
<td>14.259</td>
<td>&gt; 50%</td>
</tr>
<tr>
<td></td>
<td>23.56</td>
<td>2.0</td>
<td></td>
<td>10.694</td>
<td></td>
</tr>
<tr>
<td></td>
<td>23.56</td>
<td>2.5</td>
<td></td>
<td>8.555</td>
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</tr>
<tr>
<td></td>
<td>23.56</td>
<td>3.0</td>
<td></td>
<td>7.129</td>
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</tr>
<tr>
<td></td>
<td>25.13</td>
<td>1.5</td>
<td>10.0</td>
<td>10.934</td>
<td>&gt; 50%</td>
</tr>
<tr>
<td></td>
<td>25.13</td>
<td>2.0</td>
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<td>8.201</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25.13</td>
<td>2.5</td>
<td></td>
<td>6.560</td>
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</tr>
<tr>
<td></td>
<td>25.13</td>
<td>3.0</td>
<td></td>
<td>5.467</td>
<td></td>
</tr>
<tr>
<td></td>
<td>26.70</td>
<td>1.5</td>
<td>10.0</td>
<td>8.629</td>
<td>&gt; 50%</td>
</tr>
<tr>
<td></td>
<td>26.70</td>
<td>2.0</td>
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<td>6.473</td>
<td></td>
</tr>
<tr>
<td></td>
<td>26.70</td>
<td>2.5</td>
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<td>5.178</td>
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<td>26.70</td>
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<td>4.315</td>
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</tr>
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<td>Tetrapod</td>
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<td>8.0</td>
<td>17.824</td>
<td>&gt; 50%</td>
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<tr>
<td></td>
<td>25.13</td>
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<td>8.0</td>
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<td>3.0</td>
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<td>5.394</td>
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</tr>
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</table>

1 1 metric ton = 1000 kg.

2 Represents damage under sustained wave action of waves as high as the 1 percent wave, not the damage resulting from a few waves in the spectrum having a height $H_1 = 1.67 H_s$

3 $H_1$ = average height of highest 1 percent of waves for given time period = $1.67 H_s$

$H_1 = 1.67 (6)$

$H_1 = 10m$
Table 8-10. Volume of concrete: primary cover layer of structure trunk.

<table>
<thead>
<tr>
<th>Type of Armor Unit</th>
<th>( w_r ) (kN/m³)</th>
<th>Slope cot θ</th>
<th>Armor Layer Area per 100 m of structure (m²)</th>
<th>( W_d ) (metric tons)</th>
<th>Required Number of Armor Units, ( N_r )</th>
<th>Volume of Concrete per 100 m of structure (m³)</th>
</tr>
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<tbody>
<tr>
<td><strong>Tribar</strong></td>
<td>23.56</td>
<td>1.5</td>
<td>3733.5</td>
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<td>5.394</td>
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</table>

1 Area = \( \frac{(9.91 + \text{crest elev}) \times 100}{\sin \theta} \).

2 Numbers of units and concrete volumes determined from Figures 8-20 and 8-21, which were derived, in turn, from Figures 7-109 and 7-111.
Figure 8-20. Engineering data: tribars.
Figure 8-22. Volume of concrete required per 100 meters of structure as a function of tribar weight, concrete unit weight, and structure slope.
Figure 8-23. Number of tribars required per 100 meters of structure as a function of a tribar weight, concrete unit weight, and structure slope.
Figure 8-24. Volume of concrete required per 100 meters of structure as a function of tetrapod weight, concrete unit weight, and structure slope.
Figure 8-25. Volume of tetrapods required per 100 meters of structure as a function of tetrapod weight, concrete unit weight, and structure slope.
(b) **Secondary Cover Layer.** The weight of the secondary cover layer $W_{R}/10$ is based on the weight of a primary cover layer made of rock $W_{R}$.

\[
W_{R} = \text{weight of primary cover layer if it were made of rock}
\]

\[
W_{R}/10 = \text{weight of secondary cover layer}
\]

\[
w_{r} = \text{unit weight of rock} = 25.92 \text{ kN/m}^3
\]

\[
K_D = 4.0 \text{ for stone under nonbreaking wave conditions}
\]

\[
W_R = \frac{w_r H^3}{g K_D (S_R - 1)^3 \cot \theta}
\]

<table>
<thead>
<tr>
<th>cot $\theta$</th>
<th>$W_R$ (metric tons)</th>
<th>$W_{R}/10$ (metric tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>24.21</td>
<td>2.42</td>
</tr>
<tr>
<td>2.0</td>
<td>18.16</td>
<td>1.82</td>
</tr>
<tr>
<td>2.5</td>
<td>14.53</td>
<td>1.45</td>
</tr>
<tr>
<td>3.0</td>
<td>12.11</td>
<td>1.21</td>
</tr>
</tbody>
</table>

(c) **Thickness of Cover Layer.** Primary and secondary layers have the same thickness.

\[
r_A = n_k \Delta \left(\frac{g W_A}{w_r}\right)^{1/3}
\]

(eq. 7-121)
where

\[ r_A = \text{thickness of cover layer (m)} \]
\[ n = \text{number of armor units comprising the layer} \]
\[ W_A = \text{weight of individual armor unit (metric tons)} \]
\[ w_r = \text{unit weight of stone material (concrete or quarystone)} \]
\[ k_\Delta = \text{layer coefficient of rubble structure} \]

(d) Number of Stones Required

\[
N_R = A \cdot n \cdot k_\Delta \left(1 - \frac{P}{100}\right) \left(\frac{w_r}{g \cdot W_A}\right)^{2/3}
\]

(eq. 7-122)

\( N_R \) = number of armor units or stones in cover layer

\( A \) = area \((m^2)\)

\( P \) = porosity \((\%)\)

<table>
<thead>
<tr>
<th>Type of Armor Unit</th>
<th>Weight of Individual Stones, ( W_A ) (metric tons)</th>
<th>Armor Layer Thickness (m) When ( n = 2 ) for the Stone Unit Weights Below</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( w_r = 23.56 \text{ kN/m}^3 )</td>
<td>( w_r = 25.13 \text{ kN/m}^3 )</td>
</tr>
<tr>
<td>Trilab(^1) ( k_\Delta = 1.02 ) ( P = 54% )</td>
<td>16</td>
<td>3.84</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>3.67</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>3.49</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>3.28</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>3.05</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>2.77</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2.42</td>
</tr>
<tr>
<td>Tetrapod ( k_\Delta = 1.04 ) ( P = 50% )</td>
<td>18</td>
<td>4.07</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>3.91</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>3.74</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>3.56</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>3.35</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>3.11</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>2.82</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2.47</td>
</tr>
</tbody>
</table>

\(^1\) \( k_\Delta \) and \( P \) from Table 7-13.
(e) **Volume and Weight of Stones in Secondary Cover Layer.**

\[ A = \frac{(12.0 - 9.91)(100)}{\sin \theta} = \frac{209}{\sin \theta} = \text{area per 100 m of structure} \]

Number of stones in secondary cover layer:

\[ r_A = n \frac{k}{A} \left( \frac{g}{10 \ w_r} \right)^{1/3} \quad (W_R \ \text{in metric tons and} \ w_r = \text{unit weight of rock} = 25.92 \text{ kN/m}^3) \]

\[ n = \frac{r_A}{k} \left( \frac{10 \ w_r}{g \ W_R} \right)^{1/3} = \text{number of layers} \]

\[ N_R = A n \frac{k}{A} \left[ 1 - \frac{p}{100} \left( \frac{10 \ w_r}{g \ W_R} \right)^{2/3} \right] \]

\[ N_R = A \left[ \frac{r_A}{k} \left( \frac{10 \ w_r}{g \ W_R} \right)^{1/3} \right] \frac{k}{A} \left[ 1 - \frac{37}{100} \left( \frac{10 \ w_r}{g \ W_R} \right)^{2/3} \right] \]

\[ N_R = \frac{6.3 \ A r_A \ w_r}{g \ W_R} \]

**Volume of secondary cover layer:**

\[ V = r_A A \]

**Volume of rock in secondary cover layer:**

\[ V_R = 0.63V \]

**Weight of Rock:**

\[ w = \frac{g \ W_R}{10 \ N_R} \quad \text{or} \quad w = 0.63 \ V \ w_r \]
Table 8-11. Summary of secondary cover layer characteristics for tribars and tetrapods.

<table>
<thead>
<tr>
<th>Type of Unit</th>
<th>$w_r$ (kN/m³)</th>
<th>cot θ</th>
<th>$W_A$ (metric tons)</th>
<th>$W_R$/10 (metric tons)</th>
<th>$\Delta$ (m)</th>
<th>$A$ per 100 m of structure (m²)</th>
<th>$N_R$ per 100 m</th>
<th>Volume of Secondary Cover Layer per 100 m (metric tons)</th>
<th>Weight of Rock per 100 m (metric tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tribar</td>
<td>23.56</td>
<td>1.5</td>
<td>14.259</td>
<td>2.421</td>
<td>3.69</td>
<td>376.8</td>
<td>957</td>
<td>1390</td>
<td>2317</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>10.694</td>
<td>1.816</td>
<td>3.36</td>
<td>467.3</td>
<td>1440</td>
<td>1570</td>
<td>2615</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>8.555</td>
<td>1.453</td>
<td>3.11</td>
<td>562.7</td>
<td>2006</td>
<td>1750</td>
<td>2913</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>7.129</td>
<td>1.211</td>
<td>2.93</td>
<td>660.9</td>
<td>2663</td>
<td>1936</td>
<td>3225</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25.13</td>
<td>1.5</td>
<td>10.934</td>
<td>2.421</td>
<td>3.31</td>
<td>376.8</td>
<td>858</td>
<td>1247</td>
<td>2077</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>8.201</td>
<td>1.816</td>
<td>3.01</td>
<td>467.3</td>
<td>1290</td>
<td>1407</td>
<td>2363</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>6.560</td>
<td>1.453</td>
<td>2.79</td>
<td>562.7</td>
<td>1800</td>
<td>1570</td>
<td>2615</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>5.467</td>
<td>1.211</td>
<td>2.63</td>
<td>660.9</td>
<td>2391</td>
<td>1738</td>
<td>2896</td>
<td></td>
</tr>
<tr>
<td></td>
<td>26.70</td>
<td>1.5</td>
<td>8.629</td>
<td>2.421</td>
<td>3.00</td>
<td>376.8</td>
<td>778</td>
<td>1130</td>
<td>1884</td>
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<tr>
<td></td>
<td>2.0</td>
<td>6.473</td>
<td>1.816</td>
<td>2.72</td>
<td>467.3</td>
<td>1166</td>
<td>1271</td>
<td>2117</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>5.178</td>
<td>1.453</td>
<td>2.53</td>
<td>562.7</td>
<td>1632</td>
<td>1424</td>
<td>2371</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>4.315</td>
<td>1.211</td>
<td>2.38</td>
<td>660.9</td>
<td>2163</td>
<td>1573</td>
<td>2619</td>
<td></td>
</tr>
<tr>
<td>Tetrapod</td>
<td>23.56</td>
<td>1.5</td>
<td>17.824</td>
<td>2.421</td>
<td>4.06</td>
<td>376.8</td>
<td>1053</td>
<td>1530</td>
<td>2549</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>13.368</td>
<td>1.816</td>
<td>3.69</td>
<td>467.3</td>
<td>1582</td>
<td>1724</td>
<td>2873</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>10.694</td>
<td>1.453</td>
<td>3.42</td>
<td>562.7</td>
<td>2206</td>
<td>1924</td>
<td>3205</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>8.912</td>
<td>1.211</td>
<td>3.22</td>
<td>660.9</td>
<td>2927</td>
<td>2128</td>
<td>3545</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25.13</td>
<td>1.5</td>
<td>13.668</td>
<td>2.421</td>
<td>3.63</td>
<td>376.8</td>
<td>941</td>
<td>1368</td>
<td>2278</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>10.251</td>
<td>1.816</td>
<td>3.30</td>
<td>467.3</td>
<td>1415</td>
<td>1542</td>
<td>2570</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>8.201</td>
<td>1.453</td>
<td>3.07</td>
<td>562.7</td>
<td>1980</td>
<td>1727</td>
<td>2877</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>6.834</td>
<td>1.211</td>
<td>2.88</td>
<td>660.9</td>
<td>2618</td>
<td>1903</td>
<td>3170</td>
<td></td>
</tr>
<tr>
<td></td>
<td>26.70</td>
<td>1.5</td>
<td>10.787</td>
<td>2.421</td>
<td>3.29</td>
<td>376.8</td>
<td>853</td>
<td>1240</td>
<td>2065</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>8.091</td>
<td>1.816</td>
<td>2.99</td>
<td>467.3</td>
<td>1282</td>
<td>1397</td>
<td>2328</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>6.471</td>
<td>1.453</td>
<td>2.78</td>
<td>562.7</td>
<td>1793</td>
<td>1564</td>
<td>2605</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>5.394</td>
<td>1.211</td>
<td>2.61</td>
<td>660.9</td>
<td>2373</td>
<td>1725</td>
<td>2874</td>
<td></td>
</tr>
</tbody>
</table>

(4) Thickness of Underlayer.

Quarrystone

\[ k_\Delta = 1.00 \]
\[ P = 37\% \]
\[ w_r = 25.92 \text{ kN/m}^3 \]
\[ n = 2 \]
The equation for the volume of the first underlayer is as follows:

\[ V_1 = \left( \frac{E + 12.0}{2 \sin \theta} + \frac{E + 12.0 - \frac{r_1}{\cos \theta}}{2 \sin \theta} \right) r_1 \times 100 \text{ m} \]

(equation derived from preliminary geometry of cross section on page 8-48)

where

\( E \) = crest elevation (m above MLW)

\( r_A \) = thickness of cover layer (m)

\( r_1 \) = thickness of first underlayer (m)

\( V_1 \) = volume of first underlayer per 100 m of structure (m³)

The equation for the volume of the core per 100 m of structure is as follows:

\[ V_c = \frac{1}{2} \left( 12.0 + E - \frac{r_1}{\cos \theta} \right)^2 (1.5 + \cot \theta) (100) \]
(equation derived from preliminary geometry of cross section on page 8-48)

(5) **Volume of First Underlayer.** The volume per 100 m of structure (in thousands of m$^3$) is shown in the tabulation below.

<table>
<thead>
<tr>
<th>Armor Unit(^1) size (metric tons)</th>
<th>(\cot \theta)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>18</td>
<td>6.899</td>
</tr>
<tr>
<td>16</td>
<td>6.637</td>
</tr>
<tr>
<td>14</td>
<td>6.374</td>
</tr>
<tr>
<td>12</td>
<td>6.073</td>
</tr>
<tr>
<td>10</td>
<td>5.732</td>
</tr>
<tr>
<td>8</td>
<td>5.313</td>
</tr>
<tr>
<td>6</td>
<td>4.853</td>
</tr>
<tr>
<td>4</td>
<td>4.274</td>
</tr>
</tbody>
</table>

\(^1\) Valid for tribars and tetrapods because \(V_1\) depends only on \(\theta\) and \(r_1\) (\(r_1\) is dependent on the armor unit size, but not the type).

See Figure 8-26 for a graphic comparison of costs.
Figure 8-26. Volume of first underlayer per 100 meters of structure as a function of armor unit weight and structure slope.
(6) Volume of Core: Tribars and Tetrapods. Volume per 100 m of structure (1000 m³) is shown in the following tabulation:

<table>
<thead>
<tr>
<th>Weight of Tribar or Tetrapod (metric tons)</th>
<th>cot θ 1.5</th>
<th>cot θ 2.0</th>
<th>cot θ 2.5</th>
<th>cot θ 3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>64.179</td>
<td>78.150</td>
<td>87.399</td>
<td>95.896</td>
</tr>
<tr>
<td>16</td>
<td>64.702</td>
<td>78.730</td>
<td>88.031</td>
<td>96.583</td>
</tr>
<tr>
<td>14</td>
<td>65.227</td>
<td>79.312</td>
<td>88.664</td>
<td>97.273</td>
</tr>
<tr>
<td>12</td>
<td>65.830</td>
<td>79.980</td>
<td>89.391</td>
<td>98.063</td>
</tr>
<tr>
<td>10</td>
<td>66.512</td>
<td>80.735</td>
<td>90.213</td>
<td>98.956</td>
</tr>
<tr>
<td>8</td>
<td>67.349</td>
<td>81.662</td>
<td>91.222</td>
<td>100.054</td>
</tr>
<tr>
<td>6</td>
<td>68.269</td>
<td>82.679</td>
<td>92.330</td>
<td>101.258</td>
</tr>
<tr>
<td>4</td>
<td>69.428</td>
<td>83.960</td>
<td>93.723</td>
<td>102.773</td>
</tr>
</tbody>
</table>

See Figure 8-27 for a graphic comparison of costs.

(7) Cost Analysis: The following cost analysis will be assumed for the illustrative purposes of this problem. Actual costs for particular project would have to be based on the prevailing costs in the project area. Costs will vary with location, time, and the availability of suitable materials. Unit costs of concrete are shown in the tabulation below.

<table>
<thead>
<tr>
<th>( w_{c} ) (lb/ft³)</th>
<th>( w_{c} ) (kN/m³)</th>
<th>Cost ($ per yd³)</th>
<th>Cost ($ per m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>23.56</td>
<td>60.00</td>
<td>78.40</td>
</tr>
<tr>
<td>160</td>
<td>25.13</td>
<td>63.00</td>
<td>82.40</td>
</tr>
<tr>
<td>170</td>
<td>26.70</td>
<td>82.50</td>
<td>107.90</td>
</tr>
</tbody>
</table>
Figure 8-27. Volume of core per 100 meters of structure as a function of armor unit weight and structure slope.
(a) Cost of Casting, Handling, and Placing Tribars and Tetrapods. Cost per unit is as follows:

<table>
<thead>
<tr>
<th>Weight of Armor Unit (tons)</th>
<th>Weight of Armor Unit (metric tons)</th>
<th>cot = 1.5 and 2.0</th>
<th>cot = 2.5 and 3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Cost per Unit ($)</td>
<td>Cost per Unit ($)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cost per Unit ($)</td>
<td>Cost per Unit ($)</td>
</tr>
<tr>
<td>16</td>
<td>14.515</td>
<td>33.91</td>
<td>37.38</td>
</tr>
<tr>
<td>14</td>
<td>12.701</td>
<td>35.88</td>
<td>39.54</td>
</tr>
<tr>
<td>12</td>
<td>10.887</td>
<td>38.65</td>
<td>42.60</td>
</tr>
<tr>
<td>10</td>
<td>9.072</td>
<td>40.25</td>
<td>44.37</td>
</tr>
<tr>
<td>8</td>
<td>7.258</td>
<td>40.47</td>
<td>44.61</td>
</tr>
<tr>
<td>6</td>
<td>5.443</td>
<td>39.38</td>
<td>43.40</td>
</tr>
<tr>
<td>4</td>
<td>3.629</td>
<td>43.75</td>
<td>48.22</td>
</tr>
<tr>
<td>2</td>
<td>1.814</td>
<td>68.25</td>
<td>75.25</td>
</tr>
</tbody>
</table>

The tabulated costs are graphically presented in Figure 8-28.

(b) Rock costs. In place, when \( w_r = 25.92 \text{ kN/m}^3 \),

<table>
<thead>
<tr>
<th>Weight (tons)</th>
<th>Weight (metric tons)</th>
<th>Cost per Metric Ton ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 to 2.0</td>
<td>1.36 to 1.81</td>
<td>25.00</td>
</tr>
<tr>
<td>1.0 to 1.5</td>
<td>0.91 to 1.36</td>
<td>20.00</td>
</tr>
<tr>
<td>0.5 to 1.0</td>
<td>0.45 to 0.91</td>
<td>20.00</td>
</tr>
<tr>
<td>up to 0.5</td>
<td>up to 0.45</td>
<td>17.50</td>
</tr>
<tr>
<td>Quarry run</td>
<td>Quarry run</td>
<td>15.00</td>
</tr>
</tbody>
</table>

8-67
Figure 8-28. Costs of casting, handling, and placing concrete armor units as a function of unit weight and structure slope.

8-68
(c) **Total Cost per 100 Meters of Structure.** The following tabulation sums revetment cost by weight of tribar unit:

<table>
<thead>
<tr>
<th>Weight of Armor Unit (metric tons)</th>
<th>( w_p ) (kN/m³)</th>
<th>( c_o t )</th>
<th>Concrete Cost per 100 m of Structure¹</th>
<th>Handling Costs per 100 m of Structure¹</th>
<th>First Underlayer Cost¹</th>
<th>Secondary Cover Layer Cost¹</th>
<th>Core Cost¹</th>
<th>Total Cost¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.259</td>
<td>23.56</td>
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<td>243.53</td>
<td>57.73</td>
<td>2822.43</td>
<td>4890.93</td>
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</table>

¹ All costs are in thousands of dollars per 100 m of structure; all the intermediate steps of cost calculation are not included.

For a graphic cost comparison, see Figure 8-29.
Figure 8-29. Total cost of 100 meters of structure as a function of tribar weight, concrete unit weight, and structure slope.
The tabulation below sums cost of revetment by tetrapod unit:

<table>
<thead>
<tr>
<th>Weight of Armor Unit (metric tons)</th>
<th>( \gamma_m ) (kN/m³)</th>
<th>( \cot \theta )</th>
<th>Concrete Cost per 100 m of Structure¹</th>
<th>Handling Costs per 100 m of Structure¹</th>
<th>First Underlayer Cost¹</th>
<th>Secondary Cover Layer Cost¹</th>
<th>Core Cost¹</th>
<th>Total Cost¹</th>
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<tr>
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<td>5232.82</td>
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</tbody>
</table>

¹ All costs are in thousands of dollars per 100 m of structure.

Note that total cost given here does not include royalty costs for using tetrapods. For a graphic cost comparison, see Figure 8-30.
Figure 8-30. Total cost of 100 meters of structure as a function of tetrapod weight, concrete unit weight, and structure slope.
(d) Selection of Armor Unit, Concrete Density, and Structure Slope Based on First Cost (Construction Cost). The preceding analysis is considered the first cost of the structure. To complete the analysis, average annual maintenance and repair costs should be established for each alternative and for a range of design wave heights. Maintenance and repair costs may modify the conditions established here as the most economical based on first cost.

1. Type of unit: tribar
2. Weight of unit: 11.5 metric tons
3. Structure slope: $\cot \theta = 1.5$
4. Unit weight of concrete: 24.87 kN/m$^3$
5. Cost per 100 meter of structure: $\$3,180,000$

Stability Check

$$W = \frac{w_r \, H^3}{K_D \left(S_r - 1\right)^3 \cot \theta \, g}$$

$K_D = 10.0$

$w_r = 24.87$ kN/m$^3$

$\cot \theta = 1.5$

$$S_r = \frac{w_r}{10.05} = 2.47$$

$H = 6$ m

$$W = \frac{(24.87) \, (6)^3}{10.0 \, (2.47 - 1)^3 (1.5) (9.806)}$$

$W = 11.5$ metric tons

6. Volume of concrete per 100 m: 5794 m$^3$
7. Number of armor units per 100 m: 1288
8. Thickness of armor layer: 3.37 m
9. Volume of first underlayer per 100 m: 5988 m$^3$
10. Thickness of first underlayer: 1.52 m
11. Weight of underlayer stone: 1.15 metric tons
12. Volume of core per 100 m: 66,000 m$^3$
13. Weight of core stone: 0.00192 - 0.0575 metric tons (1.92 to 57.5 kg)
14. Volume of secondary cover layer per 100 m: 1271 m$^3$
15. Thickness of secondary cover layer: 3.37 m
16. Weight of secondary cover layer stone: 2.421 metric tons

3. **Diffraction Analysis: Diffraction Around Breakwater.**

For the purposes of this problem, establish the required breakwater length so that the maximum wave height in the harbor is 1 meter when the incident wave height is 6 meters (1 percent wave for $H_s = 3.59$ m) and the period $T = 7.78$ s. Assume waves generated in Delaware Bay.

\[
L_o = \frac{gT^2}{2\pi} = 1.56 \times T^2 = 1.56 \times (7.78)^2 = 94.42 \text{ m}
\]

Depth at breakwater $d = 16.50$ m

Depth in basin $d = 31.74$ m

\[
\frac{d}{L_o} = \frac{31.74}{94.42} = 0.33817
\]

From Appendix C, Table C-1,

\[
\frac{d}{L} = 0.34506
\]

Therefore,

$L = 91.98$ m, say $L = 92$ m
The 200-m distance, therefore, translates to
\[
\frac{y}{L} = \frac{200}{92} = 2.17
\]

At 200 meters, the wave height should be 1 meter.

\[
H_1 = K_S \quad (6)
\]

\[
1 = K_S \quad (6)
\]

\[
K_S = 0.167
\]

From Figure 7-61

\[
\frac{x}{L} = 8
\]

\[
x = (8) (92)
\]

\[
x = 736 \text{ m} \quad \text{say 750 m}
\]

required breakwater length = 750 m.

4. Preliminary Design of Quay Wall Caisson.

Since the quay will be protected by breakwaters after construction is complete, the caisson will experience extreme wave action only during construction. For illustrative purposes the following conditions will be used to evaluate the stability of the caisson against wave action. It should be noted that these conditions have a low probability of occurrence during construction.

\[
H_s = 3.59 \text{ m}
\]

\[
H_1 = 6.0 \text{ m}
\]

\[
T_s = 7.78 \text{ s}
\]

\[
d = 12.0 + 1.5 \quad \text{1}
\]

\[
d = 13.5 \text{ m}
\]

Note that the bearing area for the quay wall acting on the foundation soil may be reduced by toe scour under the edge or by local bearing capacity failures near the toe when the foundation pressure there exceeds the soil's bearing capacity.

Further information on this problem may be found in Eckert and Callender, 1984 (in press) or in most geotechnical textbooks.

\[\text{1Probability of extreme surge during construction is assumed negligible.}\]
For preliminary design, assume 75 percent voids filled with seawater and unit weight of water \( w_w = 10.05 \text{ kN/m}^3 \).

a. **Nonbreaking Wave Forces on Caisson (see Ch. 7, Sec. III.2).**

   (1) Incident Wave Height: \( H_i = 6 \text{ m} \).
   
   (2) Wave Period: \( T_s = 7.78 \text{ s} \).
   
   (3) Structure Reflection Coefficient: \( \chi = 1.0 \).
   
   (4) Depth: \( d_s = 13.5 \text{ m} \).

   \[
   \frac{H_L}{gT^2} = \frac{6}{(9.806)(7.78)^2} = 0.0101
   \]

   \[
   \frac{H_L}{d_s} = \frac{6}{13.5} = 0.444
   \]

   (5) **Height of Orbit Center Above SWL (see Fig. 7-90).**

   \[
   \frac{h_o}{H_L} = 0.37
   \]

   \[ h_o = 0.40 (H_i) = 0.37(6) = 2.22 \text{ m} \]
(6) **Height of Wave Crest Above Bottom** (see Fig. 7-88).

\[ y_c = d_e + h_o + \frac{1 + \chi}{2} H_i \]
\[ y_c = 13.5 + 2.22 + \left( \frac{1 + 1}{2} \right) \]
\[ y_c = 21.72 \text{ m} \]

Wave will overtop caisson by 1.2 meters; therefore assume structure is not 100 percent reflective. Use 0.9 and recalculate \( h_o \).

\[ \frac{h_o}{H_i} = 0.36 \]  
(see Fig. 7-93)

\[ h_o = 0.36 H_i = 0.36 (6) = 2.16 \text{ m} \]

\[ y_c = 13.5 + 2.16 + \left( \frac{1 + 0.9}{2} \right) \]
\[ y_c = 21.36 \text{ m} \]

(7) **Dimensionless Force** (Wave Crest at Structure) (see Fig. 7-94). For

\[ \frac{H_i}{g t^2} = \frac{6}{(9.806)(7.78)^2} = 0.0101 \]
\[ \frac{H_i}{d_e} = \frac{6}{13.5} = 0.444 \]
\[ \chi = 0.9 \]

\[ \frac{F}{w d_e^2} = 0.33, \quad F_c = 0.33 (10.05) (13.5)^2 = 604.48 \text{ kN/m (force due to wave)} \]

Hydrostatic force is not included.

(8) **Hydrostatic Force.**

\[ F = \frac{w d_e^2}{2} = \frac{(10.05) (13.5)^2}{2} = 915.81 \text{ kN/m} \]

(9) **Total Force.**

\[ F_t = 604.43 + 915.81 = 1520.24 \text{ kN/m} \]

(10) **Force Reduction Due to Low Height.**

\[ b = 12.0 + 8.5 = 20.50 \text{ m} \]
\[ y_c = 21.36 \text{ m} \]

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\[
\frac{b}{\gamma_c} = \frac{20.50}{21.36} = 0.9597
\]

From Figure 7-97, \( r_f = 0.998 \)

\[
F_t = 0.998 \times 1520.24 = 1517.20 \text{ kN/m}
\]

(11) **Net Horizontal Force (Due to Presence of Waves).**

\[
F_{\text{net}} = 1517.20 - 915.81 = 601.39 \text{ kN/m}
\]

(12) **Dimensionless Moment (Wave Crest at Structure) (see Figure 7-95).**

\[
\frac{H_c}{gT^2} = 0.0101, \quad \frac{H_c}{d_s} = 0.444, \text{ and } x = 0.9
\]

\[
\frac{M_c}{wd^3} = 0.24
\]

\[
M_c = 0.24 \times 10.05 \times (13.5)^3 = 5934.4 \text{ kN-m/m}
\]

(13) **Hydrostatic Moment.**

\[
M = \frac{wd^3}{6} = \frac{10.05 \times (13.5)^3}{6} = 4121.1 \text{ kN-m/m}
\]

(14) **Total Moment.**

\[
M_t = 4121.1 + 5934.4 = 10,055.5 \text{ kN-m/m}
\]

(15) **Moment Reduction for Low Height.**

From Figure 7-97 with \( \frac{b}{\gamma_c} = 0.9597 \)

\[
r_m = 0.996
\]

\[
M = 0.996 \times 10,055.5 = 10,015.3 \text{ kN-m/m}
\]

(16) **Net Overturning Moment About Bottom (Due to Presence of Waves).**

\[
M_{\text{net}} = 10,015.3 - 4121.1 = 5894.2 \text{ kN-m/m}
\]
b. Stability Computations.

(1) Overturning.

(a) Weight per Unit Length of Structure.

Concrete, \( w_r = 23.56 \text{kN/m}^3 \) (25 percent of area)

Water in voids, \( w_w = 10.05 \text{kN/m}^3 \) (75 percent of area)

Height = 20.5 m

Equation for weight/unit length:

\[
W = 20.5 L_c \left\{ (0.25)(23.56) + (0.75)(10.05) \right\}
\]

\[
W = 275.26 L_c
\]

(b) Uplift per Unit Length of Structure (see Equation 7-75 and Figure 7-89).

\[
P_1 = \frac{1 + \sqrt{\frac{w w L_c}{2}}}{2} \frac{H_L}{\cosh \left( \frac{2\pi d}{L} \right)}
\]
\[ L_o = 1.5606 \times (7.78)^2 = 94.470 \text{ m} \]
\[ \frac{d}{L_o} = \frac{13.5}{94.47} = 0.1429 \]

\[ \frac{d}{L} = 0.1773 \rightarrow L = 76.14 \text{ m} \text{ (see Table C-1)} \]

\[ \cosh \left( \frac{2\pi d}{L} \right) = 1.687 \]

\[ p_1 = \frac{1 + 0.9 \times (10.05) \times (6)}{1.687} = 33.957 \text{ kN/m}^2 \]

\[ p_2 = w_w \cdot d \text{ (hydrostatic pressure)} \]

\[ p_2 = (10.05) \times (13.5) = 135.68 \text{ kN/m}^2 \]

Equations for uplift forces/unit length:

\[ B_1 = \frac{p_1 L_c}{2} = \frac{(33.957) \times (L_c)}{2} = 16.979 \text{ L}_c \]

\[ B_2 = P_2 L_c = 135.68 \text{ L}_c \]

(2) **Summation of Vertical Forces.**

\[ B_1 + B_2 - W + R_v = 0 \]

\[ 16.979 \text{ L}_c + 135.68 \text{ L}_c - 275.26 \text{ L}_c + R_v = 0 \]

\[ R_v = 122.601 \text{ L}_c \text{ kN/m} \]

(3) **Summation of Moments About A.**

\[ B_1 \frac{2}{3} L_c + B_2 \frac{1}{2} L_c - W \frac{1}{2} L_c + R_v \frac{1}{3} L_c + M_{\text{net}} = 0 \]

\[ 16.979 \frac{2}{3} \text{ L}_c^2 + 135.68 \frac{1}{2} \text{ L}_c^2 - 275.26 \frac{1}{2} \text{ L}_c^2 + 122.601 \frac{1}{3} \text{ L}_c^2 + 5894.2 = 0 \]

\[ \text{L}_c = 18.298 \text{ m} \]

This is the width required to prevent negative soil bearing pressure under caisson (reaction within middle third). Assume \( L_c = 18.5 \text{ m} \).

\[ ^1 R_v = \text{vertical component of reaction } R \]
(4) Sliding.

Coefficient of friction (see Table 7-16) for concrete on sand

\[ \mu_s = 0.40 \]

Vertical Forces for \( L_c = 18.5 \text{ m} \)

\[
W = 275.26 \ L_c = 5092.31 \text{ kN/m} \\
B_1 = -16.979 \ L_c = -314.11 \text{ kN/m} \\
B_2 = -135.68 \ L_c = -2510.08 \text{ kN/m} \\
\sum F_v = 5092.31 \cdot 314.11 \cdot 2510.08 = 2268.12 \text{ kN/m}
\]

(5) Horizontal Force to Initiate Sliding.

\[
F_H = \mu_s \ F_v = 0.40 \cdot 2268.12 = 907.25 \text{ kN/m}
\]

Since the actual net horizontal force is only 601.39 kN/m, the caisson will not slide.

c. Caisson Stability after Backfilling.

(1) Assumptions:

(a) No wave action (protected by breakwater).
(b) Voids filled with dry sand.
(c) Minimum water level at -0.91 MLW.
(d) Surcharge of 0.6 meter on fill (dry sand).

OVERTURNING SEAWARD
(2) Earth Pressure Diagrams.

(2) EARTH PRESSURE DIAGRAMS

NOTE: \( \phi = 25^\circ \)
\[ \tan^2(45^\circ - \phi/2) = 0.406 \]

<table>
<thead>
<tr>
<th>Diagram Number</th>
<th>Force (kN/m)</th>
<th>Moment Arm (m)</th>
<th>Moment (kN \cdot m/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((0.406)(0.6)(18.85)(19.9)) = 91.378</td>
<td>(\frac{19.9}{2} = 9.95)</td>
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</tr>
<tr>
<td>2</td>
<td>((0.406)(10.21)(11.09)^2) - 254.909</td>
<td>(\frac{11.09}{3} = 3.70)</td>
<td>943.16</td>
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<tr>
<td>3</td>
<td>(\frac{10.05 (11.09)^2}{2} = 618.015)</td>
<td>(\frac{11.09}{3} = 3.7)</td>
<td>2286.66</td>
</tr>
<tr>
<td>4</td>
<td>(0.406 \frac{(18.85(8.81)^2)}{2} + 8.81(18.85)(11.09) = 1044.732)</td>
<td>7.96</td>
<td>8316.07</td>
</tr>
</tbody>
</table>

8-82
(3) Total Horizontal Earth Force.
\[ F_E = 2009.034 \text{ kN/m} \]

(4) Total Overturning Moment.
\[ M_E = 12455.10 \text{ kN} - \text{m/m} \]

(5) Moment Arm.
\[ r = \frac{M_E}{F_E} = \frac{12455.10}{2009.034} = 6.20 \text{ m} \]

(6) Weight/Unit Length.

Voids filled with dry sand:
\[ W = L_\sigma (12 + 7.9 + 0.6) \{(23.56)(0.25) + (18.85)(0.75)\} = 410.56 L_\sigma \text{ kN/m} \]

(7) Uplift Force.
\[ P_1 = wd = 10.05 (11.09) = 111.45 \text{ kN/m}^2 \]
\[ B = 111.45 L_\sigma \text{ kN/m} \]

(8) Hydrostatic Force (Seaward Side).
\[ F_H = \frac{wd^2}{2} = \frac{10.05 (11.09)^2}{2} = 618.02 \text{ kN/m} \]
(moment arm = \[ \frac{11.09}{3} = 3.70 \text{ m above bottom} \]

(9) Summation of Vertical Forces.
\[ B + R^1_v - W = 0 \]
\[ 111.45 L_\sigma + R_v - 410.56 L_\sigma = 0 \]
\[ R_v = 299.11 L_\sigma \]

(10) Summation of Moments About A.
\[ \frac{W L_\sigma}{2} + F_H (3.70) - B \frac{L_\sigma}{2} - M_E - R_v \frac{L_\sigma}{3} = 0 \]
\[ \frac{410.561}{2} L_\sigma^2 + 618.02 (3.70) - \frac{111.45}{2} L_\sigma - 12455.10 - \frac{299.11}{3} L_\sigma^2 = 0 \]
\[ 49.85 L_\sigma^2 = 10168.426 \]

\[ 1 R_v = \text{vertical component of reaction } R. \]
\[ L_c^2 = 203.98 \]
\[ L_c = 14.28 \text{ m} \]
\[ R_v = 299.11 \times 14.28 = 4271.3 \text{ kN} \]

Required width of caisson = \( L_c = 14.28 \) meters.

d. Soil Bearing Pressure.

\[ R_v = \text{VERTICAL COMPONENT OF REACTION } R \]

\[ R_v = \frac{p_{\text{max}} L_c}{2} \]

\[ p_{\text{max}} = \frac{2 R_v}{L_c} = \frac{2 \times 4271.3}{14.28} = 598.22 \text{ kN/m}^2 \]

(1) Sliding.

Summation of horizontal forces:

\[ F_E - F_h - R_H^1 = 0 \]

\[ R_H = 2009.034 - 618.02 = 1391.014 \text{ kN/m} \]

Vertical forces:

\[ R_v = 4271.3 \text{ kN/m} \]

\[ ^1 R_H - \text{horizontal component of reaction } R. \]

\[ ^2 \text{Factor of safety against sliding should be 2: hence } F_H \geq 2 R_H \text{ for safe design. Caisson should be widened.} \]
Coefficient of friction:

\[ \mu = 0.40 \]

(2) **Horizontal Force to Initiate Sliding.**

\[ F_H = \mu R_v \]

\[ F_H = 0.40 \times 4271.3 = 1708.52 \text{ kN/m} \]

\[ F_H > R_H^2 \]

Caisson will not slide.

e. **Summary.** The preceding calculations illustrate the types of calculations required to determine the stability of the proposed quay wall. Many additional loading conditions also require investigation, as do the foundation and soil conditions. Field investigations to determine soil conditions are required, in addition to hydraulic model studies to determine wave effects on the proposed island.

V. **COMPUTATION OF POTENTIAL LONGSHORE TRANSPORT**

(see Ch. 4, Sec. V)

Using the hindcast deepwater wave data from Table 8-4, the net and gross potential sand transport rates will be estimated for the beaches south of Ocean City, Maryland (see Fig. 8-31). Assume refraction is by straight, parallel bottom contours.

Azimuth of shoreline = 20 degrees

1. **Deepwater Wave Angle \( (\alpha_o) \).** The angle the wave crest makes with the shoreline (equal to the angle the wave ray makes with normal to shoreline) is shown in the following tabulation:

<table>
<thead>
<tr>
<th>Direction of Approach from North (degrees)</th>
<th>Deepwater Angle ( \alpha_o ) (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>65 ( \text{southward} )</td>
</tr>
<tr>
<td>75</td>
<td>35</td>
</tr>
<tr>
<td>105</td>
<td>5</td>
</tr>
<tr>
<td>135</td>
<td>25 ( \text{northward} )</td>
</tr>
<tr>
<td>165</td>
<td>55</td>
</tr>
<tr>
<td>195</td>
<td>85</td>
</tr>
</tbody>
</table>
Figure 8-31. Local shoreline alignment in vicinity of Ocean City, Maryland.
Table 8-12. Deepwater Wave Statistics (summary of data in Table 8-4).

<table>
<thead>
<tr>
<th>$H_s$ (m)</th>
<th>Duration for These Deepwater Wave Angles (Azimuth of Shoreline = 20°)</th>
<th>Total (hour/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_o = 65^\circ$</td>
<td>$\alpha_o = 35^\circ$</td>
</tr>
<tr>
<td>0.25</td>
<td>91</td>
<td>49</td>
</tr>
<tr>
<td>0.75</td>
<td>319</td>
<td>159</td>
</tr>
<tr>
<td>1.25</td>
<td>163</td>
<td>103</td>
</tr>
<tr>
<td>1.75</td>
<td>68</td>
<td>50</td>
</tr>
<tr>
<td>2.25</td>
<td>30</td>
<td>18</td>
</tr>
<tr>
<td>2.75</td>
<td>27</td>
<td>36</td>
</tr>
<tr>
<td>3.25</td>
<td>27</td>
<td>23</td>
</tr>
<tr>
<td>3.75</td>
<td>18</td>
<td>14</td>
</tr>
<tr>
<td>4.25</td>
<td>15</td>
<td>9</td>
</tr>
<tr>
<td>4.75</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>5.25</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>5.75</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>6.25</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6.75</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>7.25</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>7.75</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>8.25</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>8.75</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>9.25</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>9.75</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

2. **Calculation of Average $F(\alpha_o)$**.

Equations (4-54) and (4-55) will be used to calculate the potential longshore sand transport rates. Since the wave angle $\alpha_o$ in both equations represents a 30-degree sector of wave directions, equation (4-55) is averaged over the 30-degree range for more accurate representation; i.e.,

$$\overline{F}(\alpha_o) = \frac{1}{\Delta\alpha} \int_{\alpha_1}^{\alpha_2} (\cos \alpha)^{1/4} \sin 2\alpha \, d\alpha$$

$$= \pm \frac{8}{9(\Delta\alpha)} \left[ (\cos \alpha_2)^{9/4} - (\cos \alpha_1)^{9/4} \right]$$

where $\Delta\alpha = \alpha_2 - \alpha_1 = \pi/6$ and the + or - sign is determined by the direction of transport. Special care should be exercised when $0^\circ < \alpha_o < 15^\circ$ and $75^\circ < \alpha < 90^\circ$. Further discussion on the method of averaging is given in Chapter 4, Section V.3,d. The results of calculation are shown in the following tabulation and also in Figure 8-32.

<table>
<thead>
<tr>
<th>$\alpha_o$, deg.</th>
<th>$\overline{F}(\alpha_o)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>0.595</td>
</tr>
<tr>
<td>35</td>
<td>0.848</td>
</tr>
<tr>
<td>5</td>
<td>0.222 or -0.058</td>
</tr>
<tr>
<td>25</td>
<td>0.708</td>
</tr>
<tr>
<td>55</td>
<td>0.780</td>
</tr>
<tr>
<td>85</td>
<td>0.152</td>
</tr>
</tbody>
</table>

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Figure 8-32. Average $F(\alpha_o)$ for 30-degree sector.
3. **Potential Longshore Transport Computed by Energy Flux Method.**

<table>
<thead>
<tr>
<th>$H_o$ (m)</th>
<th>$\alpha_o$, $H_o$ $[10^3 \text{ m}^3/\text{year}]$ for These Deepest Wave Angles</th>
<th>65°</th>
<th>35°</th>
<th>25°</th>
<th>15°</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.392, 0.301</td>
<td>0.808</td>
<td>-0.021</td>
<td>-0.395</td>
<td>-0.635</td>
</tr>
<tr>
<td>0.75</td>
<td>21.412, 15.210</td>
<td>3.206</td>
<td>-0.818</td>
<td>-13.817</td>
<td>13.399</td>
</tr>
<tr>
<td>1.75</td>
<td>37.959, 39.779</td>
<td>10.830</td>
<td>-2.829</td>
<td>-71.738</td>
<td>-114.690</td>
</tr>
<tr>
<td>2.75</td>
<td>46.656, 88.659</td>
<td>21.921</td>
<td>-5.727</td>
<td>-133.651</td>
<td>-176.691</td>
</tr>
<tr>
<td>3.25</td>
<td>70.841, 86.006</td>
<td>18.600</td>
<td>-4.859</td>
<td>-106.149</td>
<td>-144.640</td>
</tr>
<tr>
<td>5.25</td>
<td>78.316, 62.010</td>
<td>3.247</td>
<td>-0.648</td>
<td>-20.709</td>
<td>-34.222</td>
</tr>
<tr>
<td>5.75</td>
<td>76.468, 62.276</td>
<td>4.076</td>
<td>-1.065</td>
<td>-</td>
<td>-42.962</td>
</tr>
<tr>
<td>6.25</td>
<td>67.279, 19.177</td>
<td>-</td>
<td>-</td>
<td>-17.640</td>
<td>-3.437</td>
</tr>
<tr>
<td>6.75</td>
<td>48.932, -</td>
<td>-</td>
<td>-</td>
<td>-21.382</td>
<td>-4.167</td>
</tr>
<tr>
<td>7.25</td>
<td>78.004, 55.586</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>7.75</td>
<td>46.078, 32.836</td>
<td>-</td>
<td>-</td>
<td>-30.203</td>
<td>-</td>
</tr>
<tr>
<td>8.25</td>
<td>26.937, -</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>8.75</td>
<td>31.206, -</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>9.25</td>
<td>35.856, -</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>9.75</td>
<td>40.900, -</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>996.894, 781.431</td>
<td>109.593</td>
<td>-28.632</td>
<td>-631.200</td>
<td>-1,036.277</td>
</tr>
</tbody>
</table>

1 $\alpha_o$, $H_o$ - $2.03 \times 10^6 \times f \times H_s^{5/2} \times F(\alpha_o)$ in $\text{m}^3/\text{year}$ where $f$ = numbers of hours of a specific wave (Table 8-12) divided by 8,766.

2 Negative values represent northward transport.
With a shoreline azimuth of 20 degrees,

\[
\begin{align*}
(Q\varphi)_{\text{south}} &= (996.9 + 781.4 + 109.6) \times 10^3 = 1.89 \times 10^6 \text{ m}^3/\text{year} \\
(Q\varphi)_{\text{north}} &= (28.6 + 631.2 + 1036.3 + 316.9) \times 10^3 = 2.01 \times 10^6 \text{ m}^3/\text{year} \\
(Q\varphi)_{\text{net}} &= (Q\varphi)_{\text{north}} - (Q\varphi)_{\text{south}} = 0.12 \times 10^6 \text{ m}^3/\text{year} \text{ (north)} \\
(Q\varphi)_{\text{gross}} &= (Q\varphi)_{\text{north}} + (Q\varphi)_{\text{south}} = 3.90 \times 10^6 \text{ m}^3/\text{year}
\end{align*}
\]

Note that the computed values are suspect since the net longshore transport is northward which is contrary to the field observations at the adjacent areas (Table 4-6). Except for the net transport rate, the computed values appear larger than those measured at various east coast locations. One of the possible factors that contribute to this discrepancy is the wave data used in the analysis. It is noted that hindcast wave data is for deep water at a location approximately 240 kilometers east of the shoreline of interest. Furthermore, energy dissipation due to bottom and/or internal friction is not considered in the analysis. Consequently, higher energy flux is implied in the sand transport computation.

Since the hindcast wave statistics are available at an offshore location\(^1\) approximately 10 kilometers off the shoreline of interest, analysis of longshore sand transport should be based on this new data rather than on the deepwater data listed in Table 8-4. By using the procedure shown in the preceding calculations, the potential sand transport rates below are obtained.

\[
\begin{align*}
(Q\varphi)_{\text{south}} &= 1.17 \times 10^6 \text{ m}^3/\text{year} \\
(Q\varphi)_{\text{north}} &= 0.66 \times 10^6 \text{ m}^3/\text{year} \\
(Q\varphi)_{\text{net}} &= 510,000 \text{ m}^3/\text{year} \text{ (south)} \\
(Q\varphi)_{\text{gross}} &= 1.83 \times 10^6 \text{ m}^3/\text{year}
\end{align*}
\]

VI. BEACH FILL REQUIREMENTS
(See Ch. 5, Sec. III,3)

A beach fill is proposed for the beach south of Ocean City, Maryland. Determine the volume of borrow material required to widen the beach 20 meters over a distance of 1.0 kilometers. Borrow material is available from two sources.

\(^1\) Station No. 32 (Corson et al., 1982).
1. Material Characteristics.
   a. Native Sand.
      \[ \phi_{84} = 2.51 \phi \ (0.1756 \text{ mm}) \] (see Table C-5).
      \[ \phi_{16} = 1.37 \phi \ (0.3869 \text{ mm}) \]

      Mean diameter (see eq. 5-2):
      \[ M_{\phi_n} = \frac{\phi_{84} + \phi_{16}}{2} \]
      \[ M_{\phi_n} = \frac{2.51 + 1.37}{2} = 1.94 \phi \ (0.2606 \text{ mm}) \]

      Standard deviation (see eq. 5-1):
      \[ \sigma_{\phi_n} = \frac{\phi_{84} - \phi_{16}}{2} \]
      \[ \sigma_{\phi_n} = \frac{2.51 - 1.37}{2} = 0.570 \phi \ (0.6736 \text{ mm}) \]

   b. Borrow--Source A.
      \[ \phi_{84} = 2.61 \phi \ (0.1638 \text{ mm}) \]
      \[ \phi_{16} = 1.00 \phi \ (0.500 \text{ mm}) \]

      Mean diameter (see eq. 5-2):
      \[ M_{\phi_A} = \frac{2.61 + 1.00}{2} = 1.81 \phi \ (0.285 \text{ mm}) \]
      \[ \sigma_{\phi_A} = \frac{2.61 - 1.00}{2} = 0.805 \phi \ (0.572 \text{ mm}) \]

   c. Borrow--Source B.
      \[ \phi_{84} = 3.47 \phi \ (0.0902 \text{ mm}) \]
      \[ \phi_{16} = 0.90 \phi \ (0.5359 \text{ mm}) \]

      Mean diameter (see eq. 5-1):
      \[ M_{\phi_B} = \frac{3.47 + 0.90}{2} = 2.19 \phi \ (0.219 \text{ mm}) \]
      \[ \sigma_{\phi_B} = \frac{3.47 - 0.90}{2} = 1.29 \phi \ (0.4090 \text{ mm}) \]

2. Evaluation of Borrow Materials (see Fig. 5-3).

   \[ \frac{M_{\phi_A} - M_{\phi_B}}{\sigma_{\phi_B}} = \frac{1.81 - 1.94}{0.57} = -0.228 \]

   \[ \frac{\sigma_{\phi_A}}{\sigma_{\phi_B}} = \frac{0.805}{0.57} = 1.412 \]

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From Figure 5-3, quadrant 2,

(Source A) $R_A$ (overfill ratio) = 1.10

$$\frac{M_{фB} - M_{фn}}{\sigma_{фn}} = \frac{2.19 - 1.94}{0.57} = 0.439$$

$$\frac{\sigma_{фB}}{\sigma_{фn}} = \frac{1.29}{0.57} = 2.6$$

From Figure 5-3, quadrant 1,

(Source B) $R_A$ (overfill ratio) = 1.55

Conclusion: use material from Source A.

3. **Required Volume of Fill.**

Rule of thumb: 2.5 cubic meters of native material per meter (1 cubic yard per foot) of beach width or 8.23 cubic meters per square meter of beach.

$$\text{Volume of native sand} = 20.00 \text{ m} \left(\frac{8.23 \text{ m}^3}{\text{ m}^2}\right) (1.00 \text{ km}) \times \frac{1000 \text{ m}}{\text{ km}}$$

Volume of native sand = $1.65 \times 10^5$ m$^3$

Volume from Source A = 1.10 ($1.65 \times 10^5$) = $1.81 \times 10^5$ m$^3$
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